

Occupational Sorting and the Structure of Status*

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Abstract

This paper investigates the impact of social status on occupational sorting in a two-sector matching framework. Social status depends both on occupational prestige and within-sector rank (local status). I show that the weights with which these components enter – the structure of status – play a crucial role for equilibrium sorting and argue that most likely these weights differ across occupations. The greater the relative importance of ranks in a sector, the better workers does the sector attract on average, which has implications for payoffs, wage levels and inequality, and profits. Although the equilibrium is typically inefficient, this is caused by the externalities caused by local status and occupational prestige *specifically*, rather than by status concerns *per se*.

Keywords: Status, Occupational Sorting, Occupational Prestige, Local Status, Two-Sector Matching.

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1 Introduction

Economic and sociological literatures on status tend to emphasise different aspects of status concerns. Economists put more accent on its individual, relative components¹. Sociologists, on the other hand, usually focus on its collective aspect, occupational prestige in particular². There are reasons to believe that individual status depends not only on the rank in the society en large, but also on the rank in smaller reference groups (local status, see Chapter 2 of Frank, 1985b), such as occupation. These two status components both influence and are influenced by occupational sorting. However, they differ strongly in the sign of this influence – a very strong talent-pool in a profession increases its prestige, but also means that impressing one’s peers is more difficult, which affects local status negatively.

The goal of this paper is to show that, indeed, occupational sorting – and, hence, wages and profits – depends crucially on the weight agents put on local status and occupational prestige; but also that there are good reasons to think that these weights differ across occupations. I focus on the scarce jobs case of the two-sector matching model from my earlier paper³ and extend it by allowing agents’ payoffs to depend not only on wages, but also on status rewards⁴. The status reward in each sector depends on three standard components: *local status*, *public status* and *occupational prestige*. The first two depend on agents’ individual characteristics (within sector rank and sector-specific talent, respectively), whereas the last depends on the average talent in the sector. Keeping occupational prestige constant, *status structure* – so the vector of status’ components weights – determines the spread of the status reward in a sector, but not its level.

The higher the weight of the individual components of status (local and public status) or the lower the weight of the collective component (occupational prestige), the more spread out is the status reward. As jobs are scarce and the lowest payoff in each sector is fixed at the reservation payoff, in partial equilibrium an increase in status’ spread increases the total payoff of each agent. In general equilibrium, this attracts more talent from sector two and, hence, increases talent supply in sector one and decreases it in sector two. The change in talent supply influences occupational prestige, payoff levels and inequality, wage levels and inequality, and profits. In particular, a fall in the weight of occupational prestige in sector one increases its prestige, which can, in some cases, increase the status reward of all agents in that sector. This suggests that the prestige of

¹See e.g. Frank (1984b), Bakshi and Chen (1996), Ireland (2001), Hopkins and Kornienko (2004), Hopkins and Kornienko (2009).

²Marshall (1964, p. 193) writes that “The mass of evidence suggests that occupation is generally regarded as an index of social status, probably the most important single index ...”.

³Gola (2015), which can be found on <http://www.pawelgola.com/research/>.

⁴Scarce jobs mean that there are at least as many agents as firms. To ensure that all firms are matched, I assume additionally that the lowest output that can be produced by any match is higher than the sum of firm’s and agent’s reservation payoffs.

an occupation is the result of that profession being able to attract top talent, rather than the reason for it.

Of course, as all these results hold for changes in the status structure of just one sector, they are meaningful only if status structure can indeed differ across occupations. This is likely to be the case. In Chapter 2 of [Frank \(1985b\)](#) evidence is provided that the extent to which people care about their rank in a particular group depends on the intensity of interaction with members of this group. Moreover, both [Marshall \(1964\)](#) and [Fershtman, Murphy, and Weiss \(1996\)](#) argue that the importance of occupational prestige is driven by informational constraints: specifically, by our inability to grant status based on individual achievements, which is caused by the lack of both information about them and the expertise to assess them correctly. Thus, membership of a particular group is often used as a proxy. This suggests that the individual components of status should be more important in sectors where information about individual achievements is easily available. To see whether this is indeed the case, I microfound status rewards. In this microfoundation, status is granted in face to face meetings, based on available information. I show that, indeed, local status weight depends positively on insiders' information and the frequency of meeting with one's peers, whereas the weight of occupational prestige depends negatively on the quality of outsiders' information.

I assume that status rewards depend only on agents' talent, not income, and only on the talent dimension that is specific to their sector. The former is motivated by the sociologists findings that although occupational prestige is positively correlated with both average wages and average education, the latter is more important (see e.g. [Hauser and Warren, 1997](#)). The latter is equivalent to assuming that a doctor's status depends on how good his medical skills are and an economist's status depends on his talent toward economics. As natural as this sounds, it has an important implication – namely, that status is not a zero-sum game. Thus, the average status reward in the economy depends on the assignment of agents to sectors: assigning agents to sectors in which they are highly talented makes them feel more appreciated and thus creates an additional surplus. The idea that the number of dimensions on which agents can be ranked is positively related with average self-esteem can be tracked back to Robert Nozick⁵.

Both occupational prestige and local status concerns create externalities, albeit of opposite signs: the entry of a high talent individual increases prestige of that profession and hence is good for everyone; at the same time, it pushes some incumbents down the ladder and decreases their local status rewards. Thus, typically, the stable assignment of agents to sectors is inefficient. It is worth noting that this inefficiency is driven by the

⁵[Nozick](#), (1974 [reprinted, 2006, Chapter 8]) postulated that the elimination of some dimensions on which agents can be ranked would reduce the aggregate well-being. A similar effect is true in my model, as in the symmetric case an increase in the interdependence between the talent dimensions results in a fall in average status reward (I do not show this formally, but it follows trivially from a reasoning analogous to that in Section 3 of [Gola \(2015\)](#)).

distortion of relative status rewards, not status concerns themselves: in fact, the efficient assignment is equivalent to the stable one of a matching problem with identical output functions, but zero weights on local status and occupational prestige. Hence, an output maximising assignment is inefficient as well, as it does not take into account that, as status is not a zero-sum game, total status surplus depends on the assignment. Finally, recalling my microfoundation, to ensure efficiency we need both perfect information and equal frequency of meetings with peers and outsiders – neither of which seems particularly likely.

As mentioned earlier, changes to status structure that result in a greater spread of sector one status reward increase talent supply in sector one and decrease it in sector two. This increases wages in sector two, but its impact on sector one wages is ambiguous: the increased talent supply has a negative effect, but the change in status structure means that the least talented agents might need a higher compensation for they decreased status reward. The change in the difference between globally highest and lowest wages is ambiguous, as the gap between top and bottom wages widens in sector two and shrinks in sector one. Interestingly, the impact of total payoffs (so wage plus status rewards) is much clearer: they rise in sector two and for top sector one agents, but fall for the least talented sector one workers. The gap between most and least well-off agents widens in both sectors and thus also in the entire economy. Thus, not only do changes in status structure affect wages and wage inequality, they can also affect them in the opposite direction than total payoffs. In other words, the society can become more egalitarian, even though wage inequality raises; and vice versa.

Smith (1776) and Fershtman and Weiss (1993) have postulated that, all other things equal, an agent's wage should be negatively related to the prestige of her occupation. The logic of their argument still holds in my model, but it turns out that status structure belongs to the things that need to be kept constant. In particular, it is possible that an increase in the spread of sector one status increases its prestige, decreases the prestige of the other occupation *and* increases all wages in sector one by more than the highest increase in sector two wages. The reason is, of course, that the increase in spread can dramatically decrease the reward from the individual status components for the least talented agents, which can outweigh the changes in occupational prestige. This could at least partially account for the mixed empirical evidence of the negative relation between wages and occupational prestige (see Fershtman and Weiss, 1993).

The fall in sector two talent supply affects negatively profits in the second industry. The impact on sector one profits is much less clear cut: they benefit from the increase in talent supply, but the least productive firms might need to compensate their workers for the decreased status rewards, which would then force the more productive firms to increase wages as well⁶. In general, the more productive firms are more likely to gain from

⁶ Otherwise, as a within-occupation move does not change an agent's status reward, the more talented

an increase in status reward spread, but both a fall and increase in profits for all firms is possible. This suggests that there is some room for profit increasing manipulations of the status structure on firms part, for example by establishing (or abolishing) industry wide rankings and awards, or promoting (or discouraging) inter-profession socialisation.

The rest of this paper is organised as follows. In the remainder of this section, I review the literature most relevant for this study. In Section 2 I set up the model, introduce status reward and characterise the unique stable assignment. In Section 3 I characterise the efficient assignment and discuss why the stable assignment is typically inefficient. In Section 4 I derive the comparative statics results for spread increasing changes in status structure. Section 5 concludes.

1.1 Related Literature

This paper is most closely related to the body of work by Chaim Fershtman, Yoram Weiss and co-authors (Weiss and Fershtman, 1992; Fershtman and Weiss, 1993; Fershtman, Murphy, and Weiss, 1996), who show that inclusion of social rewards into a general equilibrium framework can vastly enhance both the economists understanding of the effects of cultural differences on wages and growth, and the sociologists understanding of what determines occupational prestige rankings. My paper is similar in spirit, but significantly enhances their analysis by considering a setting in which talent and status are not one-dimensional; there are concerns for local status as well as occupational prestige, sectors differ in their status structures and firms are not identical.

Another closely related body of work is that by Robert Frank (especially Chapters 2 and 3 of Frank, 1985a and its companion papers Frank, 1984a and Frank, 1984b), who considers the impact of local status concerns on internal wage structures and the sorting of agents to organisations. Frank interprets local status as status within firm, the economics of which are quite different than that of within-occupation local status. A firm takes into account the effect of their hiring decisions on the well-being of other employees, and thus internalises the externalities produced by local status concerns. An occupation consists of workers employed by many independent firms, neither of which considers the effect of their hiring decisions on everyone else in that profession. Thus, within-firm relative concerns influence mostly internal wage spreads, whereas within-occupation relative concerns affect mostly occupational sorting and only indirectly wage structures.

There is a number of papers in the sorting literature concerned with the peer effect. In the model by de Bartolome (1990) families care about schooling, the quality of which depends both on peers and expenditure on schooling. Children's ability is binary in that model. Becker and Murphy (2000) also use a two-type model to discuss the implications

workers would be willing to leave for less productive firms.

of the peer effect on residential sorting. The closest to my work is the paper by [Damiano, Li, and Suen \(2010\)](#), in which ability is continuous, both peer effect and relative concerns are present, and agents sort into two organisations with fixed capacity. As ability is one-dimensional, the stable equilibrium has one of the organisations attracting all most able workers and the other one attracting all the least talented workers, with workers of medium ability joining both organisations.

Finally, this paper contributes methodologically to the multivariate matching literature, by demonstrating that the characterisation of stable matchings developed in [Gola \(2015\)](#) can be applied to settings with externalities. The presence of externalities renders the standard linear programming method (see e.g. [Chiappori, Orefice, and Quintana-Domeque, 2011](#); [McCann, Shi, Siow, and Wolthoff, 2012](#); [Lindenlaub, 2014](#)) useless, as it finds only the efficient matchings and externalities make the stable matching inefficient. For the same reason, in settings with externalities existence of the stable matching is not guaranteed by the results from [Gretsky, Ostroy, and Zame \(1992\)](#). To the best of my knowledge, my existence and uniqueness results are the first for a multivariate matching model with externalities.

2 Model

There are two populations: agents and firms. Agents have two separate skills – X and Y –, given by a bivariate distribution $F(x, y) : [x_l, x_h] \times [y_l, y_h] \rightarrow [0, 1]$, which is twice continuously differentiable and has strictly positive density for all (x, y) in the support of $F(\bullet)$. Firms are divided into two sectors, one and two. Sector $i \in \{1, 2\}$ firms have productivity Z^i , given by a univariate, strictly increasing and continuously differentiable distribution $H_{Z^i}(z^i) : [z_l^i, z_h^i] \rightarrow [0, 1]$. A match between an agent and a firm in sector one produces an *output* of $\Pi^1(x, z^1)$ and a match between an agent and a firm in sector two produces an output of $\Pi^2(y, z^2)$. Additionally, agents receive a *status reward* $T^i(x, y, \Theta)$, which depends not only on their skills and sector, but also on the *assignment* of agents to sectors (see Definition 3 below). Thus, the total surplus produced in a match is given by $S^1(x, y, z^1, \Theta) = \Pi^1(x, z^1) + T^1(x, y, \Theta)$ in sector 1 and by $S^2(x, y, z^2, \Theta) = \Pi^2(y, z^2) + T^2(x, y, \Theta)$ in sector 2. The mass of agents is normalised to 1; the mass of sector i firms is $R^i > 0$ and jobs are scarce: $R^1 + R^2 \leq 1$. Any agent and firm are free not to match anyone, in which case they produce a reservation output normalised to 0; unmatched agents receive also a reservation status reward \bar{T} . For simplicity, I assume that, for any Θ and $i \in \{1, 2\}$, $\inf \Pi^i \geq \bar{T} - \inf T^i$. This ensures that the output is always high enough to make any match preferable to remaining unmatched.

The status reward functions are specified in detail in Section 2.3. The output functions $\Pi^1 : [x_l, x_h] \times [z_l^1, z_h^1] \rightarrow \mathbf{R}^+$, $\Pi^2 : [y_l, y_h] \times [z_l^2, z_h^2] \rightarrow \mathbf{R}^+$ are assumed to be (a) twice continuously differentiable with (b) $\Pi_x^1, \Pi_y^2 > 0$, $\Pi_z^1, \Pi_z^2 \geq 0$ and (c) Π_{xz}^1, Π_{yz}^2 that are

weakly positive and such that if $\Pi_{xz}^i(x, z) > 0$ for any (x, z) , then $\Pi_{xz}^i(x', z') > 0$ for any $(x', z') > (x, z)$.

2.1 Copula Formulation

The above *original formulation* of the matching problem is not convenient to work with. Therefore, as in Gola (2015), I apply the probability integral transformation to all random variables and work with the copula of the original skill distribution most of the time⁷. I call this the *copula formulation*.

Denote the marginals of $F(x, y)$ as $F_X(x)$ and $F_Y(y)$ and define the talents $U = F_X(X)$ and $V = F_Y(Y)$. F 's unique (by Sklar's Theorem) copula is given by:

$$C(u, v) = F(F_X^{-1}(u), F_Y^{-1}(v)).$$

As $F_{xy}(\bullet) > 0$, it follows that $F_X(\cdot)$ and $F_Y(\cdot)$ are strictly increasing and differentiable, which implies that so are $F_X^{-1}(\cdot)$ and $F_Y^{-1}(\cdot)$. Given that, it follows that $C(u, v)$ is twice continuously differentiable and has strictly positive density.

Let us now apply probability integral transformation to productivities and define $H = H_{Z^1}^{-1}(Z^1) = H_{Z^2}^{-1}(Z^2)$; clearly, H is standard uniform distributed. Note that any firm type can be uniquely defined by the vector (h, i) , where i denotes the sector the firm belongs to. Therefore, whenever an agent with talent vector (u, v) is matched with a firm with productivity (h, i) , they produce a surplus of:

$$s^1(u, v, h, \Theta) = \pi^1(u, h) + \tau^1(u, v, \Theta) = \Pi(F_X^{-1}(u), H_{Z^1}^{-1}(h)) + T^1(F_X^{-1}(u), F_Y^{-1}(v), \Theta),$$

$$s^2(u, v, h, \Theta) = \pi^2(v, h) + \tau^2(u, v, \Theta) = \Pi(F_Y^{-1}(v), H_{Z^2}^{-1}(h)) + T^2(F_X^{-1}(u), F_Y^{-1}(v), \Theta),$$

for $i = 1$ and $i = 2$, respectively. We can easily see that the output functions $\pi^1(\bullet)$ and $\pi^2(\bullet)$ inherit from $\Pi^1(\bullet)$ and $\Pi^2(\bullet)$ properties (a) to (c). I call the tuple $\{\pi^1(\bullet), \pi^2(\bullet)\}$ the *output structure*.

2.2 (Stable) Matchings and Assignments

The basic objects of my analysis are analogous to those in Gola (2015). Note that all the definitions – and generally the remainder of this section, unless stated otherwise – refer to the copula formulation.

Definition 1. A *matching* consists of a subset of matched agents $A_A \subset [0, 1] \times [0, 1]$, a subset of matched firms $A_F \subset [0, 1] \times \{1, 2\}$ and a matching function, $\zeta : A_A \rightarrow A_F$.

⁷Here this is even more natural, as the status rewards are going to depend on ranks only.

Any matching needs to satisfy a ‘measure consistency’ property, which requires that the mass of any subset of matched firms is equal to the subset of agents they are matched with (see Legros and Newman, 2002, p. 929). Formally, for any measurable set $B \subset A_F$ define its partition into the set of all sector one firms: $B^1 = \{(h, 1) \in B\}$ and all sector two firms: $B^2 = \{(h, 2) \in B\}$.

Definition 2. A matching is *measure consistent* if for any measurable $B \subset A_F$ and its preimage $\zeta^{-1}(B) \subset A_A$:

$$\int \int_{\zeta^{-1}(B)} C_{uv}(u, v) \, du \, dv = R^1 \int_{B^1} 1 \, dh + R^2 \int_{B^2} 1 \, dh.$$

Most of the time my focus will be on the way in which workers are assigned to sectors. This information can be easily inferred from a matching.

Definition 3. Any matching $(A_A, A_F, \zeta(\bullet))$ results in an *assignment* Θ , given by the set A_A and an assignment function $\theta : A_A \rightarrow \{1, 2\}$:

$$\theta(u, v) = \begin{cases} 1 & \text{if } \zeta(u, v) \in A_F^1 \\ 2 & \text{if } \zeta(u, v) \in A_F^2. \end{cases}$$

Define a *payoff scheme* as a pair of mappings: $w : [0, 1]^2 \rightarrow \mathbf{R}^+$ and $r : [0, 1] \times \{0, 1\} \rightarrow \mathbf{R}^+$. The first mapping – w – will be interpreted as wages and the second – r – as profits. Note that the total payoff agents receive is equal to the sum of their wage and status reward. As any unmatched firm and agent produce zero output and the sum of the firm’s profit and the worker’s wage in any given match cannot exceed the output they produce, we can define payoff schemes that are feasible for a given matching.

Definition 4. Given a matching, any associated payoff scheme is *feasible* iff:

$$\begin{aligned} & \text{for all } (u, v, h, i), \text{ such that } \zeta(u, v) = (h, i) : \quad w(u, v) + r(h, i) \leq \pi^i(u, v, h) \\ & \text{for all } (u, v) \notin A_A \quad w(u, v) = 0 \\ & \text{for all } (h, i) \notin A_F \quad r(h, i) = 0. \end{aligned}$$

We can also define stable matchings, so measure consistent matchings in which there exists no agent-firm pair that would prefer to be assigned with each other rather than with their current matches.

Definition 5. A matching $(A_A, A_F, \zeta(\bullet))$, which results in an assignment Θ , is *stable* if and only if it is measure consistent and there exists a payoff scheme that is feasible given $(A_A, A_F, \zeta(\bullet))$, such that for any (u, v, h, i) :

$$w(u, v) + \tau^i(u, v, \Theta) + r(h, i) \geq s^i(u, v, h, \Theta). \quad (1)$$

An assignment $\Theta = (A_A, \Theta(\bullet))$ is *stable* if and only if there exists a stable matching that results in $(A_A, \Theta(\bullet))$.

2.3 Status Rewards

The only substantial difference between the model from Gola (2015) and this one is the introduction of a status reward. Following Frank (1985a) and Marshall (1964), my status reward function consists of three components: *local status*, *public status* and *occupational prestige*⁸. Status depends only on the talent coordinate relevant for production in that sector – so the u -coordinate in sector one and the v -coordinate in sector two.

For a given assignment Θ , denote the marginal distribution of U among sector one workers as $G^1(\cdot, \Theta)$ and its mean as $\bar{u}(\Theta)$; similarly, the marginal distribution of V among sector two workers is $G^2(\cdot, \Theta)$ and its mean $\bar{v}(\Theta)$. Then the status reward functions are given by:

$$\tau^1(u, \Theta) = \left[l^1(2G^1(u, \Theta) - 1) + (1 - p^1)(2u - 1) + p^1(2\bar{u}(\Theta) - 1) \right] k,$$

$$\tau^2(v, \Theta) = \left[l^2(2G^2(v, \Theta) - 1) + (1 - p^2)(2v - 1) + p^2(2\bar{v}(\Theta) - 1) \right] k,$$

where $k > 0$, $l^i \geq 0$ and $p^i \in [0, 1]$. The first term, within-sector rank, represents local status. The second term, talent, stands for public status. The last term, the average talent in the sector, is interpreted as occupational prestige. Note that, even though the weights of status components differ across sectors, the average status reward in each sector is always equal to occupational prestige. Thus, conditional on occupational prestige, the weights determine the spread of the status reward, rather than its level (which depends on parameter k and is the same in both sectors). The vector $\{l^i, p^i\}$ denotes the *status structure in sector i* .

Frank (1985a) discusses at length why the relative importance of local status depends on the intensity of contacts within the reference group. Marshall (1964) and Fershtman et al. (1996) note that the better the information about an individuals' achievements and talents, the less important occupational prestige is, relative to local and public status⁹. Both these effects, but the latter especially, strongly suggest that status components' weights vary across occupations, as sectors differ in how much interaction with co-workers is required and how well are achievements publicised. In order to provide some structure

⁸Frank uses the term 'global status' instead of public status and 'halo effect' instead of occupational prestige. Marshall only uses the term occupational prestige, but he describes both local and public status without naming them.

⁹For example, Marshall (1964, p. 181) writes that "[...] a person may be recognised as a representative of a particular group or social class. It is obvious that it is only in terms such as these that we can speak about the social status of a group, for instance of teachers. But an individual teacher may, by virtue of personality and attributes not characteristic of the group, acquire a rather different social status within a community in which he is well known."

for those loose observations, I will next microfound the status rewards.

2.3.1 Microfoundation

My microfoundation is based on the premise that status originates from face-to-face meetings¹⁰. Whenever a matched agent – a worker – meets someone, she is ranked by that person, based on the available information¹¹. If the worker is ranked high by the other person, she receives a positive utility; if she is ranked low, she receives a negative utility. I assume that the ranking is based on the talent dimension relevant for the worker’s sector: sector one workers are ranked based on the u-coordinate of talent and sector two workers based on the v-coordinate¹².

Matched agents meet people both during and after work. During work, they meet only agents from the same sector; after work, they are equally likely to meet anyone. Sectors differ both in the quality of information about workers’ talents and the frequency of work-meetings; the number of meetings after work is the same for both sectors and normalised to one. The information the worker’s peers have about her achievements is at least as good as the information of the outsiders. A detailed description of the information structure and the way in which status is awarded can be found in Appendix A, together with derivations.

In such a framework, in any stable assignment the expected status reward takes the form stated above, with $l^i = f^i + (n^i - o^i)R^i$ and $p^i = 1 - o^i$, where $o^i \in [0, 1]$ stands for the quality of outsiders’ information, $n^i \in [o^i, 1]$ for the quality of insiders’ information and $f^i \geq 0$ is the number of work-meetings¹³. Hence, the importance of local status depends positively on the quality of insiders’ information, frequency of work-meetings and the size of the sector; and negatively on the quality of outsiders’ information. These relations are consistent with what the literature postulates, but it is worth elaborating on the negative impact of outsiders information on local status. Even in after work meetings, insiders are able to rank the worker more precisely than outsiders – hence, it is more important how one ranks against peers than outsiders. However, as outsiders information improves, the difference between their and insiders’ information falls, which decreases the relative

¹⁰See Chapter 2 of [Frank \(1985a\)](#) for a review of evidence that status comparisons are intensified by face-to-face meetings.

¹¹The exact assumptions about the status of unmatched workers are not crucial, as long as they result in a common status reward for all unemployed agents. The most natural way to achieve that is to assume no information about unmatched agents’ talents, which should be approximately true for most talent dimensions.

¹²Note that this implies that if the ranking exercise is mutual, it is possible that both agents receive positive status utility. Thus, status is not a zero-sum game in my model.

¹³More generally, in non-stable assignments, $l^i = f^i + (n^i - o^i)M^i$, where M^i is the mass of sector i workers. Thus, technically, this microfoundation implies that the weight on local status depends on the assignment; however, as for measure consistent assignments $\tau^1(u^c)$ is lowest for $M^1 = R^1$, in any stable assignment the mass of sector one agents is fixed at R^1 (see Section 2.4.2). Same reasoning applies for sector two.

importance of local status. Nevertheless, the spread of the status reward increases, as more people than before are able to work precisely.

2.4 Characterisation Strategy

To characterise the stable matching in this model I use the same two step strategy as in Gola (2015). First, I fix the assignment and consider each sector in partial equilibrium, which allows me to find the within-sector stable matchings and associated payoffs. Then, I use those payoff functions to characterise the stable assignment.

2.4.1 First Step

Fix and suppress Θ , let the *critical abilities* u^c and v^c be the upper lower bounds of $G^1(\cdot)$ and $G^2(\cdot)$ supports, respectively, and denote the mass of agents in sector i as $M^i \leq R^i$ (by measure consistency). Define the within-sector matching functions as $\zeta^1(u) : [u^c, 1] \rightarrow [0, 1]$, $\zeta^2(v) : [v^c, 1] \rightarrow [0, 1]$. In particular:

Definition 6. The *positive, assortative within-sector matching functions (PAM)* are: $P^1(u) = \frac{1}{R^1}(R^1 + M^1(G^1(u) - 1))$ and $P^2(v) = \frac{1}{R^2}(R^2 + M^2((G^2(v) - 1))$.

Then the following result holds:

Proposition 1. All feasible wage schemes that can support a stable within-sector matching (meet Inequality 1) are of the following form:

$$w^1(u) = \int_{u^c}^u \pi_u^1(r, P^1(r))dr + C^1, \quad (2)$$

$$w^2(v) = \int_{v^c}^v \pi_v^2(r, P^2(r))dr + C^2, \quad (3)$$

where $C^1 \in [\bar{T} - \tau^1(u^c), \pi^1(u^c, \frac{R^1 - M^1}{R^1})]$ and $C^2 \in [\bar{T} - \tau^2(v^c), \pi^2(v^c, \frac{R^2 - M^2}{R^2})]$.

Proof. For any sector i agent and any sector i firm, Inequality 1 takes the form:

$$w(u, v) + r(h, i) \geq \pi^i(u, v, h)$$

and thus, as only one of the talent dimensions matters for output, the expressions for $w^1(u)$ and $w^2(v)$ follow from standard results¹⁴. $C^1 + \tau^1(u^c)$ cannot be less than \bar{T} , as u^c would prefer to be unmatched. C^1 cannot be greater than $\pi^1(u^c, \frac{R^1 - M^1}{R^1})$, because of measure consistency, wage feasibility and the fact that $\pi^1(u, \cdot)$ is non-decreasing. $\pi^1(u^c, \frac{R^1 - M^1}{R^1}) \geq \bar{T} - \tau^1(u^c)$ follows from the fact that $\inf_{u,h} \pi^1 \geq \bar{T} - \inf_u \tau^1$ for any Θ . \square

¹⁴For strictly supermodular outputs from Sattinger (1979), for weakly supermodular outputs from Proposition 1 in Gola (2015).

As agents' status does not depend on the specific firm they are matched with, but only on the sector, firms still have to pay competitive wages, given the distribution of talent in the industry¹⁵.

2.4.2 Second Step

Proposition 1 holds for all assignments, including the stable ones. In this part, I will use this to derive further conditions that hold in stable assignments and, hence, I suppress Θ from notation again.

As jobs are scarce, and any match is preferable to remaining unmatched, it follows from measure consistency that all firms end up matched¹⁶. Hence, $M^1 = R^1$ and $M^2 = R^2$ – sector sizes are fixed. Note that this implies that $P^i(\cdot) = G^i(\cdot)$, so the firm an agent is matched with depends only on her sectoral rank. By definitions of u^c and v^c the mass of matched agents with $(u, v) < (u^c, v^c)$ has to be zero. At the same time, any agent with $u > u^c$ or $v > v^c$ will receive a payoff strictly greater than \bar{T} ¹⁷. Therefore, all agents with $(u, v) < (u^c, v^c)$ and only such agents will remain unmatched and thus:

$$C(u^c, v^c) = 1 - R^1 - R^2. \quad (4)$$

Moreover, it has to be the case that agents with (u^c, v^c) need to earn identical payoffs in both sector, as otherwise some agents would want to relocate.

Lemma 1. In any stable assignment $w^1(u^c) + \tau^1(u^c) = w^2(v^c) + \tau^2(v^c)$. Additionally, if $R^1 + R^2 < 1$, then $w^1(u^c) + \tau^1(u^c) = \bar{T}$.

If one of the sectors offers higher wages at the top end of the distribution, it attracts all the agents who are highly talented in both dimensions. Define the *star abilities* u^* and v^* :

$$\begin{aligned} u^* = 1 \quad \text{and} \quad w^1(1) + \tau^1(1) = w^2(v^*) + \tau^2(v^*), \quad & \text{if } w^1(1) + \tau^1(1) \leq w^2(1) + \tau^2(1) \\ v^* = 1 \quad \text{and} \quad w^1(u^*) + \tau^1(u^*) = w^2(1) + \tau^2(1), \quad & \text{if } w^1(1) + \tau^1(1) > w^2(1) + \tau^2(1). \end{aligned}$$

Hence, stability requires that any agent with $u \in (u^*, 1]$ joins sector one and any agent with $v^* \in (v^*, 1]$ joins sector two. Note also that it is always the case that

$$\max\{u^*, v^*\} = 1. \quad (5)$$

¹⁵The possibility that agents' status depends also on the firm they are matched with is very real and interesting, but outside the scope of this paper.

¹⁶Suppose not. Then, by measure consistency we have positive masses of unmatched firms and agents. This contradicts stability, as for any Θ and $i \in \{1, 2\}$, $\inf \Pi^i \geq \bar{T} - \inf T^i$ and the output functions are strictly increasing in talent.

¹⁷By Proposition 1 and the facts that output is strictly increasing in talent and status reward nondecreasing.

Define the set $\Gamma = [u^c, u^*] \times [v^c, v^*]$. As $R^1, R^2 > 0$, Γ is non-empty¹⁸. Agents with $(u, v) \in \Gamma$ will join sector one if $w^1(u) \geq w^2(v)$ and sector two if $w^2(v) \geq w^1(u)$. Define a function $\psi(v) : [v^c, v^*] \rightarrow [u^c, u^*]$ such that:

$$w^1(\psi(v)) + \tau^1(\psi(v)) = w^2(v) + \tau^2(v). \quad (6)$$

Any agent in Γ strictly prefers sector one if $u > \psi(v)$ and sector two if $u < \psi(v)$.

Remark 1. The quintuple $(u^c, v^c, u^*, v^*, \psi(\bullet))$ fully defines the stable assignment, as in [Gola \(2015\)](#). Agents with $(u, v) < (u^c, v^c)$ are unmatched. Sector 1 is populated by agents with: (i) $v < v^c$ and $u > u^c$; (ii) $(u, v) \in \Gamma$ and $\psi(v) < u$; (iii) $u > u^*$. Sector 2 is populated by agents with: (i) $v > v^c$ and $u < u^c$; (ii) $(u, v) \in \Gamma$ and $\psi(v) > u$; (iii) $v > v^*$. The set of remaining agents is of zero mass.

Thus, given $(u^c, v^c, u^*, v^*, \psi(\bullet))$, we can derive the marginal distributions of talent in each sector (details can be found in Appendix B.1):

$$G^1(u) = \begin{cases} 0 & \text{for } u < u^c, \\ \frac{1}{R^1} \int_{u^c}^u C_u(r, \psi^{-1}(r)) dr & \text{for } v \in [u^c, u^*], \\ G^1(u^*) + \frac{1}{R^1}(u - u^*) & \text{for } u > u^*. \end{cases} \quad (7)$$

$$G^2(v) = \begin{cases} 0 & \text{for } v < v^c, \\ \frac{1}{R^2} \int_{v^c}^v C_v(\psi(r), r) dr & \text{for } v \in [v^c, v^*], \\ G^2(v^*) + \frac{1}{R^2}(v - v^*) & \text{for } v > v^*. \end{cases} \quad (8)$$

Substitute these talent distributions into the wage and status reward functions, then it follows from Equation (6) that, for $v \in [v^c, v^*]$:

$$\begin{aligned} & \int_{u^c}^{\psi(v)} \pi_u^1 \left(t, \int_{u^c}^t \frac{C_u(r, \psi^{-1}(r))}{R^1} dr \right) + 2k \left(l^1 \frac{C_u(t, \psi^{-1}(t))}{R^1} + (1 - p^1) \right) dt \\ &= \int_{v^c}^v \pi_v^2 \left(t, \int_{v^c}^t \frac{C_v(\psi(r), r)}{R^2} dr \right) + 2k \left(l^1 \frac{C_v(\psi(t), t)}{R^2} + 2(1 - p^2) \right) dt. \end{aligned} \quad (9)$$

$G^1(1)$ and $G^2(1)$ have to equal 1, which implies:

$$\int_{u^c}^{u^*} C_u(r, \psi^{-1}(r)) dr + 1 - u^* = R^1, \quad (10)$$

$$\int_{v^c}^{v^*} C_v(\psi(r), r) dr + 1 - v^* = R^2. \quad (11)$$

¹⁸As $R^1, R^2 > 0$, it has to be the case that $v^c, u^c < 1$. Agents' payoff functions are strictly increasing and therefore, by Lemma 1, $w^2(1) + \tau^2(1) > w^1(u^c)$ and $w^1(1) + \tau^1(1) > w^2(v^c) + \tau^2(v^c)$, which implies that $v^* > v^c$ and $u^* > u^c$.

A solution to Equations (4), (5) and (9)-(11) gives us $(u^c, v^c, u^*, v^*, \psi(\bullet))$ and thus fully characterises a stable assignment. Note that each of these equations needs to hold for any stable assignment and thus any stable assignment has to be represented by a solution to Equations (4), (5) and (9)-(11).

Theorem 1. A solution to Equations (4), (5) and (9)-(11) exists, is unique and fully characterises the unique stable assignment as specified in Remark 1.

The main idea behind the proof of this result (see Appendix B.2) is identical to that behind the existence and uniqueness proofs in Gola (2015). I define a map, the fixed point of which is equivalent to the solution of Equation (9) and find a norm for which this map is a *contraction mapping*¹⁹. This proves that $\psi(\cdot)$ is unique *given* (u^c, v^c) – and also continuous in these two variables. Then showing existence and uniqueness is merely a matter of proving that the remaining equations have a unique solution given the function $\psi(u^c, v^c)$. Note that, as there are externalities in this model, existence does not follow from the standard results in Gretsky et al. (1992). In fact, I am not aware of any other existence results for multivariate matching with externalities.

Trivially, the existence of a stable assignment implies existence of stable matchings.

Corollary 1. A matching in which agents are assigned to sectors as specified in Theorem 1 and are positively and assortatively matched within sectors is always stable. Moreover, if the surplus functions in each sector are strictly supermodular for all possible agent-firms pairs, then this is the only stable matching.

This follows from Theorem 1 and the proof of Proposition 1 in Gola (2015).

3 Efficiency

The presence of local status concerns and occupational prestige creates externalities, as the decision of any agent to join a given sector affects the surplus produced by all the matches in that industry. Other than in Frank (1985a), these externalities are not internalised, as each occupation consists of many independent and infinitely small firms, and there is no-one to regulate the entry of agents to sector. Thus, the stable and efficient assignments are not necessarily equivalent and the question of efficiency needs to be investigated separately.

Definition 7. A matching $(A_A, A_F, \zeta(\bullet))$ is *efficient* if and only if it is measure consistent and the total surplus produced under any measure consistent matching $(A'_A, A'_F, \zeta(\bullet)')$ is weakly lower than the total surplus produced under $(A_A, A_F, \zeta(\bullet))$ ²⁰.

¹⁹The norm I use is Bielecki's norm for a high-enough parameter λ .

²⁰Define a function $s^i(u, v, h, i, \Theta)$, which is equal to $s^1(u, h, \Theta)$ for $i = 1$ and to $s^2(v, h, \Theta)$ for $i = 2$. Then the total surplus produced under $(A_A, A_F, \zeta(\bullet))$ is given by $\int \int_{A_A} s(u, v, \zeta(u, v), \Theta) du dv$, where Θ is the assignment that results from $(A_A, A_F, \zeta(\bullet))$.

Thus, a matching is efficient if it maximises the surplus produced in the whole economy (see e.g. [Becker, 1973](#); [Gretsky et al., 1992](#); [Chiappori, McCann, and Nesheim, 2010](#); [Lindenlaub, 2014](#)).

Proposition 2. The efficient assignment is unique and equivalent to the stable assignment of a matching problem with identical output structure, but zero weights on local status and occupational prestige.

Proof. For any assignment Θ , the most efficient matching that results in Θ is positive and assortative within sectors²¹. The average status reward is always equal to $\bar{u}(\Theta)k$ in sector one and to $\bar{v}(\Theta)k$ in sector two, neither of which depends on the respective status' structures. These two facts combined imply that the total surplus produced in the economy under assignment Θ does not depend on the status components' weight and neither does efficiency. If $l^i = p^i = 0$, then surplus functions do not depend on the assignment and the results from [Gretsky et al. \(1992\)](#) apply. Hence, stable assignments are efficient and efficient assignments are stable. As, by Theorem 1, the stable assignment is unique, the uniqueness of the efficient assignment follows. \square

In the absence of local status and occupational prestige, there are no externalities and the stable assignment is efficient. Local status and occupational prestige create an inefficiency as they distort the relative rewards of agents: local status rewards the highly ranked agents too much, whereas occupational prestige does not reward them enough. As the total surplus produced in the economy does not depend on the structures of status, in order to find the efficient assignment it suffices to find the stable assignment for the case of no local status and occupational prestige concerns (see Section 2.4).

The inefficiency is caused by the distortion of relative status rewards, rather than status concerns *per se*. If the social planner tried to ignore status concerns and assigned agents to sectors in a way that maximises total output, she would also create an inefficiency. The reason is that status is not a zero-sum game in this model and thus the total status reward in the economy depends on the assignment. In other words, the social planner could make some potentially great artists feel inefficiently unappreciated, if, in her quest for maximal output, she assigned them to sciences or engineering. In fact, even if local status and occupational prestige matter and the stable assignment is inefficient, it is still possible that its total surplus will be higher than in the output maximising assignment.

If we recall the microfoundation of the status reward functions, it becomes clear that the real culprits are imperfect information and the fact that agents care more about the opinion of their peers, as these are the reasons why individual status rewards depend on

²¹This is the case, as once we fix the assignment, the model becomes equivalent to two separate Becker-Sattinger industries with surplus functions given by $\pi^1(u, h) + \tau^1(u, \Theta)$ and $\pi^2(v, h) + \tau^2(v, \Theta)$; and in a Becker-Sattinger industry PAM is both stable and efficient.

local status and occupational prestige. In fact, in order to ensure efficiency of the stable matching, we need *both* perfect information about achievements and talents, and equal chances of meeting insiders and outsiders, neither of which is likely at all. This does not mean that the stable assignment is necessarily inefficient: local status and occupational prestige create externalities of opposite signs (and occur in both sectors), so it can, by pure luck, happen that these effects cancel each other out²². Nevertheless, other than in [Fershtman et al. \(1996\)](#), perfect information on its own is not enough to ensure efficiency, as it eliminates only occupational prestige, not local status.

4 Changes to Status Structure

In this section, I investigate how changes to the status structure affect occupational sorting and through that payoffs, wages and profits. As I have shown in my earlier work, in two sector matching models the spread of surplus is of crucial importance, especially if jobs are scarce. The more spread out surplus is, the weaker the competition between agents, which increases talent supply. This reasoning works equally well with status concerns – and as surplus’ spread depends on the spread of status reward, we should expect status structure to play an important role.

Consider two matching problems, the *old* and the *new*, which meet conditions (a)-(c) from Section 2, as well as the requirement that every match is worthwhile. To formally distinguish between them, I introduce a parameter – ρ ; the old problem is denoted by ρ_1 and the new one by ρ_2 . For example, $u^c(\rho_2)$ is the new critical ability in sector one, whereas $\Theta_s(\rho_1)$ is the old stable assignment in sector two. Throughout this section, I assume that the second sector’s status and output structures are the same in the old and new problems.

To define a more spread out status reward, I use the notion of spread introduced by [Bickel and Lehmann \(1979\)](#).

Definition 8. A distribution $F_X(x, \rho_2)$ is *more spread out in Bickel-Lehman sense* than distribution $F_X(x, \rho_1)$ if:

$$F_X^{-1}(u_2, \rho_2) - F_X^{-1}(u_1, \rho_2) \geq F_X^{-1}(u_2, \rho_1) - F_X^{-1}(u_1, \rho_1) \text{ for all } 0 \leq u_1 < u_2 \leq 1. \quad (12)$$

If there exists some x such that Equation (12) holds strictly for all $x \leq u_1 < u_2 \leq 1$, then distribution $F_X(x, \rho_2)$ is *more spread out, strictly from the x th quintile*.

As the status reward function $T(X, \Theta)$ depends on the assignment, our definition of *status reward spread* needs to specify for which assignments does status reward become

²²To see this, consider symmetric sectors. Then agents sort into the sector in which they are more talented, which is the efficient outcome.

more spread out. For the results below to hold, it is sufficient that status reward becomes more spread out for the old stable assignment $\Theta(\rho_1)$.

Definition 9. Sector one status reward becomes (*strictly*) *more spread out* if the new distribution of status reward $W = T(X, \Theta(\rho_1))$ is more spread out in Bickel-Lehman sense (strictly from the $u^c(\rho_1)$ th quintile) than the old distribution of W .

Both an increased weight of local status and a lower weight of occupational prestige make status depend more on the individual talent, rather than the sectoral average and thus strictly spread status reward out²³. These could be caused by improvements in information about talents and achievements – for insiders, outsiders or both – or by an increase of within-occupation interaction and socialisation (recall Section 2.3.1).

Theorem 2. If sector one status reward becomes strictly more spread out, then (i) the distribution of talent in sector one improves in first order stochastic dominance sense and (ii) the distribution of talent in sector two deteriorates in first order stochastic dominance sense. The prestige of occupation one strictly increases and the prestige of occupation two falls strictly.

Proof. If sector one status reward becomes strictly more spread out, so does sector one surplus, and all results follow immediately from Theorem 3 in Appendix C. \square

The intuition is very simple. Scarcity of jobs implies that the reservation payoff in sector one is fixed. Hence, ignoring reallocation, a strictly more spread out status reward increases payoffs for all sector one agents. In general equilibrium this attracts additional talent from sector two, increases talent supply in sector one and decreases it in sector two. In other words, the more does status depend on individual achievements in a sector, the more attractive is that sector for highly talented people.

The link between surplus' spread and occupational prestige is also quite interesting, as it implies that the less important occupational prestige is, the more prestigious the occupation. More generally, the prestige of an occupation is not the reason why that profession is able to attract top talent, but its result. This, in turn, is more likely if the information about individual achievements is readily available. Given that this is largely the case in academia, at least for insiders (think of Hirsch-index, citations or ideas.repec rankings), it could be part of the reason why top talent still joins academia, despite lower wages (see [Weiss and Lillard, 1978](#)).

The negative effects of surplus' spread on the status reward of lowly ranked agents are at least partially mitigated by the resulting increase in occupational prestige. In fact, it is possible that the general equilibrium effect of higher occupational prestige dominates the direct impact of spread, if the stable assignment is sensitive to small changes in spread

²³An increase in the importance of status, k , makes status reward more spread out in both sectors and thus the results below do not hold for it.

and the weight on occupational prestige is high²⁴. As the wages paid to the least talented agents depend on their status reward, it follows that the direction of change in sector one wage levels is ambiguous. Much more can be said, however, about sector two wages, as well as total payoffs in both sectors.

To simplify the following discussion on wages and payoffs I focus on the case of strictly scarce jobs ($R^1 + R^2 < 1$); note that all results hold also for $R^1 + R^2 = 1$ if we assume that $C^1 + \tau^1(u^c)$ does not change and, additionally, that the surplus function in sector two is strictly supermodular.

Proposition 3. If jobs are strictly scarce and sector one status reward becomes strictly more spread out, then (i) both wages and total payoffs strictly increase for all agents in sector two; (ii) in sector one, total payoffs strictly increase for a positive mass of the most talented agents and strictly fall for a positive mass of the least talented agents; (iii) the gap between top and bottom total payoffs increases strictly in both sector and the whole economy, but (iv) the gap between top and bottom wages shrinks strictly in sector one, strictly widens in sector two and its change in the whole economy is ambiguous.

Proof. Firstly, note that Theorem 2 implies that v^c decreases and u^c increases. In fact, for strictly scarce jobs both of these changes are strict, which follows from Theorem 3 in Appendix C.

(iv) For any $v'' > v' \geq v^c$ we have:

$$w^2(v'') = \int_{v'}^{v''} \pi_v^2(r, G^2(r)) dr + w^2(v'). \quad (13)$$

A deterioration in sector two talent distribution in first order stochastic dominance sense implies that $G^2(r)$ increases for all r and therefore, as surplus is supermodular, $w^2(v'')$ increases more than $w^2(v')$. The strict fall in v^c implies that $w^2(v^c)$ increases by strictly more than C^2 and thus the difference between $w^2(1)$ and C^2 increases strictly. An analogous reasoning holds for sector one, except that there talent distribution improves and, hence, $G^1(r)$ decreases for all r and u^c strictly increases. Clearly, the change in the difference between the overall highest and overall lowest wage can go both ways.

(i) As v^c falls strictly, so does the public status reward of the least talented sector two agent. Since the local status of the lowest ranked agent is always equal to -1 and occupational prestige falls strictly, it follows that $\tau(v^c)$ decreases strictly. Thus, by Lemma 1 the lowest wage $w^2(v^c) = C^2$ increases strictly and the strict increase in wages follows from the proof of (iv).

The public status reward remains unchanged for agent of any talent. Thus, the change in total payoff depends on the change in local status reward, occupational prestige and

²⁴I discuss the conditions needed for that in Appendix C.1 in detail, but in general the spread of output in sector one and surplus in sector two need to be low compared to k .

wage. Local status reward increases by Theorem 2 and the increase in wage has to be strictly greater than the fall in occupational prestige reward²⁵. Therefore, total payoff increases strictly.

(ii) Denote sector i total payoffs as $t^i(\cdot)$. I start at the top. Theorem 2 and the proof of Lemma 6 (Appendix C) imply that $u^*(\rho_2) \leq u^*(\rho_1)$ and $v^*(\rho_2) \geq v^*(\rho_1)$. Thus:

$$\begin{aligned} t^1(u^*(\rho_1), \rho_2) &\geq t^1(u^*(\rho_2), \rho_2) = t^2(v^*(\rho_2), \rho_2) \geq t^2(v^*(\rho_1), \rho_2) \\ t^2(v^*(\rho_2), \rho_1) &\geq t^2(v^*(\rho_1), \rho_1) = t^1(u^*(\rho_1), \rho_1) \geq t^1(u^*(\rho_2), \rho_1) \end{aligned}$$

which trivially implies that:

$$t^1(u^*(\rho_1), \rho_2) - t^1(u^*(\rho_1), \rho_1) \geq t^2(v^*(\rho_2), \rho_2) - t^2(v^*(\rho_2), \rho_1). \quad (14)$$

Thus, $t^1(u^*(\rho_1))$ strictly increases. For any $u > u^*$ we have that:

$$t^1(u) = \int_{u^*}^u \pi_u^1(r, G^1(r)) + 2(l^1 - p^1 + 1) \, dr + w(u^*(\rho_1)). \quad (15)$$

For $u > u^*(\rho_1)$, $G^1(u)$ does not change; and as strict spread of status reward implies that l^1 or $1 - p^1$ strictly increase, it follows that $t^1(u, \rho_2) > t^1(u, \rho_1)$ for any $u \in [u^*(\rho_1), 1]$.

I will turn now to payoffs of the least talented agents. Recall that u^c strictly increases. As total payoff strictly increases in talent, it follows from definition of critical ability that $t^1(u^c(\rho_2), \rho_1) > t^1(u^c(\rho_1), \rho_1) = 0 = t^1(u^c(\rho_2), \rho_2)$. Existence of a positive mass of agents for whom wages decrease (increase) follows from continuity of wage functions.

(iii) Follows from (i), (ii) and the fact that with strictly scarce jobs the lowest payoff is fixed at the reservation status. \square

The effect on agents' payoffs is similar to that of a spread in surplus on wages in [Gola \(2015\)](#). In sector two, talent supply falls and therefore payoffs increase. Sector one payoffs are affected positively by the increase in status reward spread and negatively by the higher supply of talent. For agents with highest talent, the effect of the increase in spread dominates and their payoffs raise. For the least talented agents the increase in talent supply dominates: those agents are pushed down the ladder so much that their payoffs fall.

As the increase in status reward spread increases top payoffs in both sectors and the

²⁵To see the latter, note that as v^c fell strictly, the increase in $w^2(v^c(\rho_1))$ is strictly greater than the increase in C^2 , which in turn is at least as great as the fall in occupational prestige reward. Formally, for any $v \geq v^c(\rho_1)$:

$$\begin{aligned} w^2(v, \rho_2) - w^2(v, \rho_1) &\geq w^2(v^c(\rho_1), \rho_2) - w^2(v^c(\rho_1), \rho_1) > C^2(\rho_2) - C^1(\rho_1) \\ &= -2k(v^c(\rho_2) - v^c(\rho_1) + \bar{v}(\rho_2) - \bar{v}(\rho_1)) > -2k(\bar{v}(\rho_2) - \bar{v}(\rho_1)). \end{aligned}$$

lowest payoffs are fixed at the reservation payoff, the gap between best and worst off workers increases. The change in wage inequality between top and bottom earners is, however, ambiguous – and definitely decreases in sector one, because of the increased talent supply. This has two important implications. Firstly, it means that changes in status structure can have an impact not only on wages levels, but also inequality. Secondly, it suggests that with status concerns, wage inequality does not tell the entire story, as it is possible that wage inequality falls (rises), despite a rise (fall) in overall inequality.

The existing literature predicts a negative relationship between the difference in occupational prestige and the difference in wages, so that, all things equal, agents should earn less in more prestigious occupation (see Book I, Chapter X, Part I in (Smith, 1776) or (Fershtman and Weiss, 1993)). This general principle holds here too, but the structures of status belong to the things that need to be equal. As status reward spreads out, the local and public status rewards fall for the least talented agents, which might not be fully compensated by the increase in occupational prestige.

Proposition 4. It is possible that, as a result of a sufficiently more spread out status reward in sector one, the lowest increase in sector one wage is greater than the highest increase in sector two wage, despite the fact that sector one becomes more and sector two less prestigious.

Proof. The change in wages in each sector depends on the change in wage constant C^i and the change in wage spread, which for sector one is given by $\int_{u^c}^u \pi_u^1(r, G^1(r)) dr$. Suppose that both the maximal spread of output, measured by $\pi^i(1, 1) - \pi^i(0, 0)$, and the maximum status reward $l^i k + p^i k$, are arbitrarily small compared $\pi^i(0, 0) - \bar{T}$, for $i \in \{0, 1\}$. Suppose further that the weight of local status increases. Then the increase in C^2 is bounded from above by $2k$ and the increase in C^1 is bounded from below by $2k(l^1(\rho_2) - l^1(\rho_1) - 1)$. Note that wage spread in each sector is bounded from below by 0 and from above by the maximal spread of output. Thus, the increase in wage in sector two is bounded from above by $2k + \pi^2(1, 1) - \pi^2(0, 0)$ and the increase in wage in sector one is bounded from below by $2k(l^1(\rho_2) - l^1(\rho_1) - 1) + \pi^1(1, 1) - \pi^1(0, 0)$. It follows that for high enough $l^2(\rho_2)$ the lowest increase in sector one wage (for $w^1(1)$) is greater than the highest increase in sector two wage (for $w^2(1)$). □

Thus, changes in status structure can result in an increase in both the prestige of an occupation and the wages earned by its members. This can explain to some extent the mixed empirical evidence for the negative link between wages and occupational prestige, as reported in Fershtman and Weiss (1993).

As changes in status structure have a strong impact on occupational sorting, it is natural to ask whether this can be used to cheaply attract more talent to a sector,

for example by creating rankings or establishing awards. This question is, in general, out of the scope of this paper – nevertheless, the following result suggests that status structure manipulations could, in certain circumstances, lead to an increase in profits in the industry in which they took place.

Proposition 5. If jobs are strictly scarce and sector one status reward becomes strictly more spread out, then (i) profits fall for all sector two firms and (ii) sector one profits can both fall or increase for all firms, but more productive firms always gain more (or lose less).

Proof. (i) The profit function for all matched firms in sector i is given by (Sattinger, 1979, see):

$$r^i(h) = \int_0^h \pi_h^i((G^i)^{-1}(r), r) dr + C_P^i, \quad (16)$$

where the profit constant C_P^i is equal to the difference between the output produced by the worst match ($\pi^i(x^c, 0)$) and the wage constant C^i . A v^c falls and C^2 increases, C_P^2 has to fall. Thus, it follows from Theorem 2 and inspection of Equation (16) that profits fall strictly in sector two.

(ii) Equation (16) implies that for any $h'' > h'$, we have:

$$r^i(h) = \int_{h'}^{h''} \pi_h^i((G^i)^{-1}(r), r) dr + r^i(h'),$$

which, by Theorem 2 and supermodularity of surplus functions, means that $r^1(h'', \rho_2) - r^1(h'', \rho_1) \geq r^1(h', \rho_2) - r^1(h', \rho_1)$ and thus the more productive sector one firms gain more than the less productive ones. In general, there are two channels through which sector one profits are affected: the improvement in talent-pool and the ambiguous change in profit constant. The fact that all sector one profits can increase follows immediately from the fact that C^1 can fall (see Appendix C.1). To see that all sector one profits can fall, suppose that both the maximal spread of output, measured by $\pi^i(1, 1) - \pi^i(0, 0)$, and the maximum status reward $l^1k + p^1k$, are arbitrarily small compared to $\pi^i(0, 0) - \bar{T}$, for $i \in \{0, 1\}$. Then a sufficiently large increase in l^i will decrease the profit constant by more than the maximum possible increase in profits spread (by a reasoning analogous to the proof of Proposition 4). \square

The fall in talent supply in sector two has a negative impact on profits through two channels. Firstly, it increases the competition for talent. Secondly, it decreases the status reward of the least talented agents, which makes the outside option more attractive and forces firms to pay them more. In sector one, on the other hand, the increased talent supply has a positive effect on profits: by decreasing competition and making the outside option less desirable. The increase in status reward spread itself, however, has a negative effect on the lowest status rewards and thus on firm profits. The end effect is ambiguous

for all firms, but the more productive ones will always gain more, or at least lose less, than the less productive ones.

5 Conclusions

The individual and collective aspects of status have very different implications for occupational sorting. Therefore, the relative importance of these components – the structure of status – can influence the way in which agents self-select into sectors. Moreover, as the existence of local status concerns and taste for occupational prestige are likely driven by informational constraints, as well as the asymmetry in meetings with peers and outsiders, there are good reasons to believe that status structure differs across sectors. I have shown that in my two-sector matching model, an increase in the weight of the individual components or a fall in the weight of the collective component in some sector result in an increase in talent supply in that industry and a fall in talent supply in the other sector. This has, in turn, important implications for total payoffs, wage levels and inequality, and profits in both sectors. I have also demonstrated that the inefficiency of the stable assignment of agents to sectors is caused by local status concerns and taste for occupational prestige, not by status concerns *per se*.

The specific implications of changes in status structure for talent supply, wages and profits might depend on my assumptions, the chief of which is that jobs are scarce. Without this assumption the model becomes much less tractable: in particular, the method used in this paper to prove existence and uniqueness would yield only existence results with abundant jobs – the number of equilibria is an open question. These issues are partly addressed in the companion paper (Gola, 2017), but that paper focuses does not provide comparative statics results.

Another issue that should be addressed in future research is the relative weighting of status dimensions by society; in this paper, they are valued equally highly. An asymmetric, but still exogenous weighting would not complicate the model too much, but neither would add much to our understanding. Allowing for asymmetric and endogenous weightings could, however, help us better understand how output and status structures influence the relative importance of talent dimensions.

Appendix

A Microfoundation of Status Rewards

I keep the assignment Θ fixed throughout this section and suppress it from notation. The distribution of talent among sector i agents is $G^i(u, v)$, and its marginals are G_V^i and G_U^i .

After agents self-select into sectors, each draws one Judge from all agents, so simply from the copula $C(u, v)$, and f^i Judges from her sector, so from $G^i(u, v)$. Denote the talent vector of the Judge as (u^J, v^J) . The Judge observes the sector in which the agent works and some signal about her ability. Then she uses this information to establish how likely it is that the agent ranks higher and, based on that, grants her some positive or negative status utility. The status utility received by an agent (u, v) from a Judge (u^J, v^J) is given by:

$$\tau(u, v, u^J, v^J) = \begin{cases} [\Pr(u \geq u^J) - \Pr(u < u^J)]k & \text{if } \theta(u, v) = 1, \\ [\Pr(v \geq v^J) - \Pr(v < v^J)]k & \text{if } \theta(u, v) = 2. \end{cases} \quad (17)$$

Information is modelled in the simplest possible way. The true talent of an agent from sector i is observed by a Judge from the same sector with probability n^i , whereas with probability $1 - n^i$ only the agent's sector is observed. If the Judge is from the other sector, then the agents talent is observed with probability o^i and with probability $1 - o^i$ only her sector is observed. Insiders have better information than outsiders:

$$n^i \geq o^i.$$

Suppose that $M^1, M^2 > 0$ ²⁶. I first derive the expected utility from non-work meetings. Suppose that $\theta(u^J, u^J) = i$ and $\theta(u, v) = 2$. The Judge observes the agents ability with probability o^2 if $i = 1$ and n^2 if $i = 2$ and grants her utility p if $v \geq v^J$ and $-p$ if $v^J > v$. The probability of drawing a Judge who is better than the agent is $G_V^i(v)$. However, with probability $(1 - o^2)$ or $(1 - n^2)$ – depending on her sector – the Judge will not observe the actual ability and only infer it from the fact that the agent works in sector two; in which case the probability that the Judge is better (worse) than the agent is $G_V^2(v^J)$. Finally, the probability of drawing a Judge from sector one is M^1 and the probability of drawing a Judge from sector two is M^2 . Thus the expected status utility for a sector two agent from after work meetings is given by:

$$\begin{aligned} \tau_a^2(v) = & \left[M^1 [o^2(2G_V^1(v) - 1) + (1 - o^2) \int_0^1 (1 - 2G_V^1(v))g_V^1(v) dv] \right. \\ & \left. + M^2 [n^2(2G_V^2(v) - 1) + (1 - n^2) \int_0^1 (1 - 2G_V^2(v))g_V^2(v) dv] \right] k. \end{aligned} \quad (18)$$

As every agent has to join either sector one or two, we have

$$M^1 G_V^1(v) + M^2 G_V^2(v) = v.$$

²⁶This is always the case in any stable matching.

Note also that $\int_{y_i}^{y_h} (1 - 2G_V^2(v))g_V^2(v) dv$ is equal to zero. Thus we can write:

$$\begin{aligned}\tau_a^2(v) &= \left[o^2(2(v - M^2 G_V^2(v)) - M^1) + M^2 n^2(2G_V^2(v) - 1) \right. \\ &\quad \left. + M^1(1 - o^2) \left[1 - [2G_V^2(v)G_V^1(v)]_0^1 + 2 \int_0^1 G_V^1(v)g_V^2(v)dv \right] \right] k \\ &= \left[(n^2 - o^2)M^2(2G_V^2(v) - 1) + o^2(2v - 1) + (1 - o^2)[2\bar{v} - 1] \right] k.\end{aligned}\quad (19)$$

The expected status utility derived from work-meetings is, by analogous reasoning, given by:

$$\tau_w^2(v) = f^2 n^2(2G_V^2(v) - 1),$$

and so the total expected status reward is equal to:

$$\tau^2(v) = \left[(n^2 f^2 + (n^2 - o^2))M^2(2G_V^2(v) - 1) + o^2(2v - 1) + (1 - o^2)[2\bar{v} - 1] \right] k.$$

And as the problem is symmetric, the status reward in sector one is given by:

$$\tau^1(u) = \left[(n^1 f^1 + (n^1 - o^1))M^1(2G_U^1(u) - 1) + o^1(2u - 1) + (1 - o^1)[2\bar{u} - 1] \right] k.$$

B Stable Assignments

Proof of Lemma 1. This follows trivially for $R^1 + R^2 < 1$, as then $1 - M^1 - M^2 > 0$ and thus $C^1 + \tau^1(u^c) = C^2 + \tau^2(v^c) = \bar{T}$, by Proposition 1. Therefore I will focus on the cases where $R^1 + R^2 = 1$ and thus $C(u^c, v^c) = 0$. This implies that $\min\{u^c, v^c\} = 0$. Suppose that $u^c = 0$ and $C^1 + \tau^1(u^c) > C^2 + \tau^2(v^c)$; as the payoff in the second sector is continuous and $v^c < 1$, there has to exist some $\epsilon > 0$ such that for any $v \in [v^c, v^c + \epsilon]$ we have $C^1 + \tau^1(u^c) > t^2(v) + \tau^2(v)$, which means that none of these agents will join sector two – which contradicts v^c 's definition.

Now, suppose that $C^1 + \tau^1(u^c) < C^2 + \tau^2(v^c)$; suppose further that $v^c > 0$. For any $\epsilon > 0$ the mass of agents with $(u, v) \in [0, \epsilon] \times [v^c - \epsilon, v^c]$ is strictly positive – as $C_{uv}(u, v) > 0$ for all (u, v) – and all agents in this set will be working in sector one. However, by continuity of surplus and wage functions it has to be the case that there exists a small enough ϵ that for all agents in this set we have:

$$t^1(u) + \tau^1(u) + r^2(0) = t^1(u) + \tau^1(u) + s^2(v^c, 0) - C^2 - \tau^2(v^c) < s^2(v, 0),$$

which contradicts stability. Now suppose that $v^c = 0$ as well – there has to exist some $\epsilon > 0$ such that for any $u \in [0, \epsilon]$ we have $C^1 + \tau^1(u) < C^2 + \tau^2(v)$, which contradicts the definition of u^c .

The proof for the case when $v^c = 0$ is analogous. □

B.1 Talent Distributions

The probability that an agent with talent $V = v$ chooses sector two is $\Pr(\theta(U, v) = 2|v)$. For $v \in [v^c, v^*]$ the probability that a sector two agent has ability lower than v is:

$$G^2(v) = \int_{v^c}^v \frac{\Pr(\theta(U, v) = 2|r)}{R^2} dr,$$

as V 's marginal distribution is standard uniform. Consider some arbitrary agent with $v \in (v^c, v^*]$. Such an agent will be in sector 2 as long as $\psi(v) \geq u$, which implies that $\Pr(\theta(U, v) = 2|v) = C_v(\psi(v), v)^{27}$. Recalling the definitions of v^c and v^* , expression (8) follows. The talent distribution in sector one can be derived by an analogous reasoning.

B.2 Existence and Uniqueness

Proof of Theorem 1. The proof of this Theorem is very similar to the proof of Theorem 1 in Gola (2015). I start by reducing the set of equations (4), (5), (9)-(11), which will be henceforth referred to as *the original set*. Consider any $r \in [v^c, v^*]$; then, by differentiating $C(\psi(r), r)$ rearranging and integrating from v^c to v , we arrive at:

THERE IS A MISTAKE HERE: A $-C(u^c, v^c)$ IS MISSING FROM THE FOLLOWING EQUATIONS.

$$R^1 G^1(\psi(v)) + R^2 G^2(v) = C(\psi(v), v). \quad (20)$$

An analogous procedure for $C(r, \psi^{-1}(r))$ gives

$$R^1 G^1(u) + R^2 G^2(\psi^{-1}(u)) = C(u, \psi^{-1}(u)).$$

This, (10) and (11) imply that $\psi(v^*) = u^*$, which follows also from the definitions of u^* and v^* .

By differentiating Equation (9), rearranging, using Equation (20) and $o^i = 1 - p^i$ and then integrating from v^c to v (and remembering that $\psi(v^c) = u^c$) we get:

$$\psi(v) = u^c + \int_{v^c}^v \frac{\pi_v^2 \left(t, \frac{\int_{v^c}^t C_v(\psi(r), r) dr}{R^2} \right) + 2k \left(\frac{l^2}{R^2} C_v(\psi(t), t) + o^2 \right)}{\pi_u^1 \left(\psi(t), \frac{C(\psi(t), t) - \int_{v^c}^t C_v(\psi(r), r) dr}{R^1} \right) + 2k \left(\frac{l^1}{R^1} C_u(\psi(t), t) + o^1 \right)} dt.$$

This still depends on $\psi(\cdot)$, u^c , v^c and indirectly on v^* . I will eliminate v^* by extending the functions $C(\bullet)$, $C_v(\bullet)$, $C_u(\bullet)$, $\pi^1(\bullet)$ and $\pi^2(\bullet)$ in a way that allows to define an extended function $\psi^e(\cdot)$, which uniquely determines $\psi(\cdot)$. The extended functions $C^e(\bullet)$, $C_v^e(\bullet)$,

²⁷It doesn't matter whether $\psi(v) \geq u$ holds strictly, as the probability of $\psi(v) = u$ is 0 anyway.

$C_u^e(\bullet)$, $\pi^{1e}(\bullet)$ and $\pi^{2e}(\bullet)$ are defined as follows: (1) $C^e : [0, 1 + B] \times [0, 1] \rightarrow [0, 1]$

$$C^e(u, v) = \begin{cases} C(u, v) & \text{for } (u, v) \in [0, 1] \times [0, 1] \\ v & \text{for } (u, v) \in (1, 1 + B] \times [0, 1], \end{cases}$$

(2): $C_v^e(u, v) : [0, 1 + B] \times [0, 1] \rightarrow [0, 1]$

$$C_v^e(u, v) = \begin{cases} C_v(u, v) & \text{for } (u, v) \in [0, 1] \times [0, 1] \\ 1 & \text{for } (u, v) \in (1, 1 + B] \times [0, 1], \end{cases}$$

(3) $C_u^e(u, v) : [0, 1 + B] \times [0, 1] \rightarrow [0, 1]$

$$C_u^e(u, v) = \begin{cases} C_u(u, v) & \text{for } (u, v) \in [0, 1] \times [0, 1] \\ C_u(1, v) & \text{for } (u, v) \in (1, 1 + B] \times [0, 1], \end{cases}$$

(4) $\pi_v^{1e}(u, h) : [0, 1 + B] \times [0, \frac{1}{R^1}] \rightarrow \mathbf{R}^+$:

$$\pi_v^{1e}(u, h) = \begin{cases} \pi_u^1(u, h) & \text{for } (u, h) \in [0, 1]^2 \\ \pi_u^1(1, h) & \text{for } (u, h) \in (1, B] \times [0, 1], \\ \pi_u^1(u, 1) & \text{for } (u, h) \in [0, 1] \times (1, \frac{1}{R^1}], \\ \pi_u^1(1, 1) & \text{for } (u, h) \in (1, B] \times (1, \frac{1}{R^1}], \end{cases}$$

(5): $\pi_u^{2e}(v, h) : [0, 1] \times [0, 1 + \frac{1}{R^2}] \rightarrow \mathbf{R}^+$:

$$\pi_u^{2e}(v, h) = \begin{cases} \pi_v^2(v, h) & \text{for } (u, h) \in [0, 1]^2 \\ \pi_v^2(v, 1) & \text{for } (u, h) \in [0, 1] \times (1, \frac{B+1}{R^2}], \end{cases}$$

where $B = \frac{2k(\frac{1}{R^2} + o^2) + \max \pi_v^2}{o^1 + \min \pi_u^1}$. The idea behind these extensions is to get functions that will be defined also for $\psi^e(v) > 1$ and such that $C^e(\cdot, v)$, $C_v^e(\cdot, v)$, $C_u^e(\cdot, v)$, $\pi_u^{1e}(\cdot, \cdot)$ and $\pi_v^{2e}(v, \cdot)$ are Lipschitz continuous²⁸; denote their Lipschitz-constants as L^1, L^2, L^3, L^4, L^5 and L^6 respectively. The fact that, for $u > 1$, $C_u^e(\bullet)$ is not a derivative of $C^e(\bullet)$ does not

²⁸ I will do this in detail for $C_v^e(u, v)$ – the reasoning for the other two is analogous. $C_v^e(u, v)$ is clearly continuous in u . It is equally easy to see that the function $C_v^e(\cdot, v)$ is differentiable almost everywhere and its derivative is Lebesgue integrable. It is also the case that for any $(u, v) \in (1, 1 + B] \times [0, 1]$ we have:

$$C_v^e(a, v) + \int_a^1 C_{uv}^e(r, v) dr + \int_1^u 0 dr = 1,$$

which means that $C_v^e(\cdot, v)$ is absolutely continuous. Moreover, as $C^e(\bullet)$ is twice continuously differentiable and any continuous function defined on a compact set is bounded it follows that $C_{vv}^e(\cdot, v)$ is essentially bounded; and a differentiable almost everywhere, absolutely continuous function with an essentially bounded derivative is Lipschitz-continuous.

matter, as $C_u^e(\bullet)$ is clearly an extension of $C_u(\bullet)$ to $[0, 1] \times [0, 1 + B]$.

Now I can define the extended function $\psi^e(v) : [v^c, 1] \in [u^c, 1 + B]$:

$$\psi^e(v) = u^c + \int_{v^c}^v \frac{\pi_v^{2e} \left(t, \frac{\int_{v^c}^t C_v^e(\psi^e(r), r) dr}{R^2} \right) + 2k \left(\frac{l^2}{R^2} C_v^e(\psi^e(t), t) + o^2 \right)}{\pi_u^{1e} \left(\psi^e(t), \frac{C^e(\psi^e(t), t) - \int_{v^c}^t C_v^e(\psi^e(r), r) dr}{R^1} \right) + 2k \left(\frac{l^1}{R^1} C_u^e(\psi^e(t), t) + o^1 \right)} dt. \quad (21)$$

which together with:

$$1 - R^1 - R^2 = C^e(u^c, v^c), \quad (22)$$

$$R^2 = \int_{v^c}^{v^*} C_v^e(\psi(r), r) dr + 1 - v^*, \quad (23)$$

$$v^* = \sup\{v \in [v^c, 1] : \psi^e(v) \leq 1\}, \quad (24)$$

$$u^* = \psi^e(v^*) \quad (25)$$

constitutes the *modified set* of equations.

Proposition 6. The solution to the modified set of equations exists and is unique.

Proof. Define the set:

$$K = \{d \in C[0, 1] : |d(v) - 1| \leq 1 + B\},$$

where $C[0, 1]$ is the set of all continuous functions that map from $[0, 1]$. The constant function $d(v) = 1$ lies in K and hence the set is non-empty. Define a norm, $\|\cdot\|_\lambda$ on $C[0, 1]$:

$$\|h\|_\lambda = \sup_{[0, 1]} e^{-\lambda v} |h(v)|,$$

where λ is some weakly positive number. K is a complete metric space for this norm.²⁹

Endow the set $[0, 1]^2$ with the Euclidean norm and define mappings $G^2 : K \times [v^c, 1] \times [0, 1] \rightarrow [0, 1]$, $s^2 : K \times [v^c, 1] \times [0, 1] \rightarrow [0, 2k(\frac{l^2}{R^2} + o^2) + \max \pi_v^2]$, $G^1 : K \times [u^c, 1 + B] \times [v^c, 1] \times [0, 1]^2 \rightarrow [0, \frac{1}{R^1}]$, $s^1 : K \times [u^c, 1 + B] \times [v^c, 1] \times [0, 1]^2 \rightarrow [0, 2k(\frac{l^1}{R^1} + o^1) + \min \pi_u^1]$ and $T : K \times [0, 1]^2 \rightarrow K$:

$$\begin{aligned} (G^2 d)(v, v^c) &= \frac{1}{R^2} \int_{v^c}^v C_v^e(d(r), r) dr, \\ (G^1 d)(u, v, v^c, u^c) &= \frac{C^e(u, v) - R^2 (G^2 d)(v)}{R^1}, \\ (s_v^2 d)(v, v^c) &= \pi_v^{2e}(v, (G^2 d)(v, v^c)) + 2k \left(\frac{l^2}{R^2} C_v^e(d(v), v) + o^2 \right) \end{aligned}$$

²⁹If we endowed K with the sup-norm, then K would be a closed subspace of $C[0, 1]$; since $C[0, 1]$ is complete in the sup-norm, so is K . And it was shown by Bielecki (1956) that the $\|\cdot\|_\lambda$ norm is equivalent to the sup-norm for any $C[a, b]$ – and thus if K is a complete metric space for the sup-norm it is also a complete metric space for $\|\cdot\|_\lambda$.

$$(s_v^1 d)(u, v, v^c) = \pi_u^{1e} \left(u, (G^2 d)(u, v, v^c) \right) + 2k \left(\frac{l^1}{R^2} C_u^e(d(v), v) + o^1 \right)$$

$$(Td)(v, v^c, u^c) = \begin{cases} u^c & \text{for } v < v^c \\ u^c + \int_{v^c}^v \frac{(s_v^2 d)(v, v^c)}{(s_v^1 d)(d(v), v, v^c)} dt & \text{for } v \geq v^c. \end{cases}$$

These maps are well-defined, as for any $v^c \in [0, 1]$ and $d \in K$:

$$(G^2 d)(v, v^c) \leq \int_{v^c}^t \frac{1}{R^2} dr \leq \frac{1}{R^2}$$

$$(G^1 d)(u, v, v^c, u^c) \leq \frac{C(d(t), t)}{R^1} \leq \frac{1}{R^1}.$$

Note that $(Td)(\bullet)$ is continuous in v , v^c and u^c . It is also the case that for $v \geq v^c$:

$$|[(Td)(v, v^c, u^c, M^2) - 1] \leq \int_{v^c}^v B dt + |u^c - 1| \leq 1 + B,$$

and for $v < v^c$:

$$|[(Td)(v, v^c, u^c, M^2) - 1] \leq |u^c - 1| \leq 1 + B,$$

so indeed $T(K) \subset K$. Finally, it should be clear that for any v^c, u^c the restriction of any fixed point of $(Td)(\bullet)$ to $[v^c, 1]$ gives us the solution to (21) and that any solution to (21) can be easily extended into a fixed point of $(Td)(\bullet)$. Therefore, by Banach Fixed-Point Theorem, it suffices to show that there exists such a λ that for any $(v^c, u^c) \in [0, 1]^2$, $Td(\bullet)$ is a contraction wrt to the norm $\|\cdot\|_\lambda$ to show that (21) has a unique solution for any feasible (u^c, v^c) .

Let us drop (v^c, u^c) from the definition of a map (remembering that we are keeping them constant). Take any any $t \geq v^c$ and any $d_1, d_2 \in S$. For any map $(fd)(t)$ denote $(fd_1)(t) - (fd_2)(t)$ as $\Delta_d(fd)(t)$ and for any map $(fd)(d(t), t)$ denote $(fd_1)(d_1(t), t) - (fd_2)(d_2(t), t)$ as $\Delta_d(fd)(d(t), t)$. Then we have:

$$|\Delta_d(G^2 d)(t)| = \frac{1}{R^2} \left| \int_{v^c}^t C_v^e(d_1(r), r) - C_v^e(d_2(r), r) dr \right| \quad (26)$$

$$\begin{aligned} &\leq \int_{v^c}^t \frac{|C_v^e(d_1(r), r) - C_v^e(d_2(r), r)|}{R^2} dr \leq \int_{v^c}^t \frac{L_2}{R^2} |d_1(r) - d_2(r)| dr \\ &= \frac{L_2}{R^2} \int_{v^c}^t e^{\lambda r} e^{-\lambda r} |d_1(r) - d_2(r)| dr \leq \frac{L_2}{R^2} \|d_1 - d_2\|_\lambda \int_{v^c}^t e^{\lambda r} dr \\ &= \frac{L_2}{R^2 \lambda} \|d_1 - d_2\|_\lambda (e^{\lambda t} - e^{\lambda v^c}) \leq \frac{L_2}{R^2 \lambda} \|d_1 - d_2\|_\lambda e^{\lambda t}, \end{aligned}$$

$$|\Delta_d(s_v^2 d)(t)| \leq |\pi_v^{2e}(t, (G^2 d_1)(t)) - \pi_v^{2e}(t, (G^2 d_2)(t))| \quad (27)$$

$$\begin{aligned} &+ \frac{2kl^2}{R^2} |C_v^e(d_1(t), t) - C_v^e(d_2(t), t)| \\ &\leq L_6 |\Delta_d(G^2 d)(t)| + \frac{2kl^2 L_2}{R^2} |d_1(t) - d_2(t)| \end{aligned}$$

$$\leq \frac{L_6 L_2}{R^2 \lambda} \|d_1 - d_2\|_\lambda e^{\lambda t} + \frac{2kl^2 L_2}{R^2} |d_1(t) - d_2(t)|$$

which can be used to establish:

$$\Delta_d(G^1 d)(d(t), t) = \left| \frac{C^e(d_1(v), v) - C^e(d_2(v), v) - R^2 \Delta_d(G^2 d)(v)}{R^1} \right| \quad (28)$$

$$\begin{aligned} &\leq \frac{1}{R^1} (|C^e(d_1(v), v) - C^e(d_2(v), v)| + |R^2 \Delta_d(G^2 d)(v)|) \\ &\leq \frac{L_2}{\lambda R^1} \|d_1 - d_2\|_\lambda e^{\lambda t} + \frac{L^1}{R^1} |d_1(t) - d_2(t)|, \\ \Delta_d(s_v^1 d)(d(t), t) &\leq |\pi_u^{1e}(d_1(t), (G^1 d_1)(d_1(t), t)) - \pi_u^{1e}(d_2(t), (G^1 d_2)(d_2(t), t))| \quad (29) \\ &\quad + \frac{2kl^1}{R^1} |C_u^e(d_1(t), t) - C_u^e(d_2(t), t)| \\ &\leq |\pi_u^{1e}(d_1(t), (G^1 d_1)(d_1(t), t)) - \pi_u^{1e}(d_1(t), (G^1 d_2)(d_2(t), t))| \\ &\quad + |\pi_u^{1e}(d_1(t), (G^1 d_2)(d_2(t), t)) - \pi_u^{1e}(d_2(t), (G^1 d_2)(d_2(t), t))| \\ &\quad + \frac{2kl^1 L_3}{R^1} |d_1(t) - d_2(t)| \\ &\leq \frac{L_2 L_5}{\lambda R^1} \|d_1 - d_2\|_\lambda e^{\lambda t} + \left(\frac{L_1 L_5 + 2kl^1 L_3}{R^1} + L_4 \right) |d_1(t) - d_2(t)| \end{aligned}$$

Denote $\sup \pi_v^2(v, h) + \frac{2kl^2}{R^2} + o^2 = L_7$, $\inf \pi_u^1(u, h) + o^1 = L_8$ and note that continuity of π_u^1 and π_v^2 and the fact that $\pi_u^1 > 0$ imply that both L_7 and L_8 are finite. Using all this, we can write, for any $v \geq v^c$ and any $d_1, d_2 \in S$:

$$\begin{aligned} |\Delta_d(Td)(v)| &= \left| \int_{v^c}^v \frac{(s_v^2 d_1)(t)}{(s_u^1 d_1)(d_1(t), t)} - \frac{(s_v^2 d_2)(t)}{(s_u^1 d_2)(d_2(t), t)} dt \right| \\ &= \left| \int_{v^c}^v \frac{(s_v^2 d_1)(t)}{(s_u^1 d_1)(d_1(t), t)} - \frac{(s_v^2 d_2)(t)}{(s_u^1 d_1)(d_1(t), t)} \right. \\ &\quad \left. + \frac{(s_v^2 d_2)(t)}{(s_u^1 d_1)(d_1(t), t)} - \frac{(s_v^2 d_2)(t)}{(s_u^1 d_2)(d_2(t), t)} dt \right| \\ &\leq \int_{v^c}^v \left(\frac{L_1 L_5 L_7 + 2kl^1 L_3 L_7}{R^1 L_8^2} + \frac{L_7 L_4}{L_8^2} + \frac{2kl^2 L_2}{L_8 R^2} \right) |d_1(t) - d_2(t)| \\ &\quad + \left(\frac{L_6 L_2}{L_8 R^2 \lambda} + \frac{L_2 L_5 L_7}{\lambda R^1 L_8^2} \right) \|d_1 - d_2\|_\lambda e^{\lambda t} dt \\ &\leq \frac{1}{\lambda} \left(\frac{L_1 L_5 L_7 + 2kl^1 L_3 L_7}{R^1 L_8^2} + \frac{L_7 L_4}{L_8^2} + \frac{2kl^2 L_2}{L_8 R^2} \right) \|d_1 - d_2\|_\lambda e^{\lambda v} \\ &\quad + \frac{1}{\lambda} \left(\frac{L_6 L_2}{L_8 R^2 \lambda} + \frac{L_2 L_5 L_7}{\lambda R^1 L_8^2} \right) \|d_1 - d_2\|_\lambda e^{\lambda t}. \end{aligned}$$

For $v < v^c$ this has to hold as well, as then $|(Td_1)(v) - T(d_2)(v)| = 0$. Denote $\frac{L_1 L_5 L_7 + 2kl^1 L_3 L_7}{R^1 L_8^2} + \frac{L_7 L_4}{L_8^2} + \frac{2kl^2 L_2}{L_8 R^2} = L_9$, then, for any $v \in [0, 1]$, we have that:

$$\Delta_d(Td)(v) \leq \frac{1}{\lambda} \|d_1 - d_2\|_\lambda e^{\lambda v} \left(\frac{L_6 L_2}{L_8 R^2 \lambda} + \frac{L_2 L_5 L_7}{\lambda R^1 L_8^2} + L_9 \right).$$

Dividing both sides of that by $e^{\lambda v}$ we get:

$$e^{-\lambda v} |\Delta_d(Td)(v)| \leq \frac{1}{\lambda} \|d_1 - d_2\|_\lambda \left(\frac{L_6 L_2}{L_8 R^2 \lambda} + \frac{L_2 L_5 L_7}{\lambda R^1 L_8^2} + L_9 \right)$$

which, by taking sup on both sides implies that:

$$\|(Td_1)(t) - T(d_2)(t)\|_\lambda \leq \frac{1}{\lambda} \|d_1 - d_2\|_\lambda \left(\frac{L_6 L_2}{L_8 R^2 \lambda} + \frac{L_2 L_5 L_7}{\lambda R^1 L_8^2} + L_9 \right). \quad (30)$$

Therefore, there has to exist a high enough λ for which our map $(Td)(v)$ is a contraction in the metric space $(S, \|\cdot\|_\lambda)$ – which, by Banach’s Fixed-Point Theorem means that $(Td)(v)$ has a unique fixed point, which in turn means that Equation (21) has a single solution for any given $(v^c, u^c) \in [0, 1]^2$. Note that Equation (30) does not depend on (v^c, u^c) – and thus, by standard results (see e.g. [Hasselblatt and Katok, 2003](#), p. 68) it follows that, as $(Td)(v, v^c, u^c)$ is continuous in v^c and u^c , the fixed point – and thus the solution of (21) – is continuous in them as well.

Denote the fixed point of $(Td)(\cdot, v^c, u^c)$ as $d^*(\cdot, v^c, u^c)$ – then the following result holds:

Lemma 2. Keeping the other parameter constant, $d^*(\cdot, v^c, u^c)$ is weakly decreasing in v^c and weakly increasing in u^c , for all v ’s, with these relations holding strictly for some v ’s.

Proof. I start with the claims regarding $d(v, \cdot, u^c)$. To simplify notation, I drop u^c from all functions, as I keep them constant for the proof of the claims regarding v^c . Take any $v_2^c > v_1^c \in [0, 1]$ and denote $d^*(v, v_2^c) - d^*(v, v_1^c)$ as $\Delta_{v^c} d^*(v, v^c)$ and:

$$G^2(v, v^c) = \frac{1}{R^2} \int_{v^c}^v C_v(d^*(r, v^c), r) dr,$$

$$G^1(d^*(v, v^c), v^c) = \frac{C(d^*(v, v^c), r)}{R^1} - G^2(v, v^c).$$

As $d^*(\cdot, v^c)$ is strictly increasing, for any $v \in [v_1^c, v_2^c)$ we have $\Delta_{v^c} d^*(v, v^c) < 0$, which proves the second (strict) part of this claim. Thus, we only need to show now that $\Delta_{v^c} d^*(v, v^c) \leq 0$ for all $v \in [v_2^c, 1]$. Suppose not. Then the set $\Omega^{gen} = \{v \in [v_2^c, 1] : \Delta_{v^c} d^*(v, v^c) > 0\}$ has to be non-empty. Define $v^g = \inf \Omega^{gen}$, then $\Delta_{v^c} d^*(v^g, v^c) = 0$ and

$\Delta_{v^c} d_v^*(v^g, v^c) > 0$. Thus, the sign of $\Delta_{v^c} d_v^*(v^g, v^c)$ depends only on the signs of³⁰:

$$\pi_v^{2e}(v^g, G^2(v_2^c, v^g)) - \pi_v^{2e}(v^g, G^2(v_1^c, v^g)) \quad (31)$$

and

$$\pi_u^{1e}(d^*(v^g, v_1^c), G^1(v_1^c, d^*(v^g, v_1^c))) - \pi_u^{1e}(d^*(v^g, v_2^c), G^1(v_2^c, d^*(v^g, v_2^c))). \quad (32)$$

However, as $\Delta_{v^c} d^*(v^g, v^c) = 0$ and both surplus functions are weakly supermodular, these in turn depend only on the sign of $G^2(v_2^c, v^g) - G^2(v_1^c, v^g)$. As for any $v \leq v^g$ it was the case that $\Delta_{v^c} d^*(v^g, v^c) \leq 0$ and $v_2^c \geq v_1^c$, it follows that: $G^2(v_2^c, v^g) - G^2(v_1^c, v^g) \leq 0$ and thus $\Delta_{v^c} d_v^*(v, v^c) \leq 0$, which means that Ω^{gen} has to be empty and proves our first claim.

As for the claim regarding u^c , note that for a change in u^c , $\Delta_{u^c} d^*(v^c, u^c)$ is positive. The subsequent reasoning is analogous, but with opposite signs (the strict decreasingness follows from $\Delta_{u^c} d^*(v^c, u^c) < 0$ and continuity). \square

Note that this Lemma and Equation (24) imply that $v^*(v^c)$ is strictly increasing in v^c and strictly decreasing in u^c . I will finish the proof by considering separately the cases of strictly and weakly scarce jobs.

Strictly scarce jobs $R^1 + R^2 < 1$ implies $C(u^c, v^c) > 0$. As for $(u, v) > 0$, $C(\bullet)$ is strictly increasing in both parameters, its inverse with respect to both parameters exists, which allows us to define u^c as a strictly decreasing, continuous function of v^c . Define \underline{v} as $u^c(\underline{v}) = 1$ and note also that as $u^c \in [0, 1]$ Equation (22) shrinks the range of feasible v^c 's to $[\underline{v}, 1]$. All of this implies that $d^*(v, v^c, u^c)$ depends only on v and v^c and is decreasing and continuous in v^c – hence, I will denote it as $d^*(v, v^c)$ from now on. Note that $d^*(v, v^c)$ uniquely determines v^* , which is strictly increasing and continuous in v^c . Thus, the modified system of equations reduces to:

$$R^2 = \int_{v^c}^{v^*(v^c)} C_v^e(d^*(r, v^c), r) + 1 - v^* dr.$$

Let us start with existence. The RHS is continuous in v^c , as $d^*(v, v^c)$ and $v^*(v^c)$ are continuous in v^c . For $v^c = \underline{v}$, we have $d^*(v, v^c) \geq 1$ regardless of v and therefore $\int_0^1 C_v^e(d^*(r, v^c), r) dr = 1$, whereas for $v^c = 1$, we have $\int_1^1 C_v^e(d^*(r, v^c), r) dr = 0$; thus,

³⁰To see this, note that:

$$\begin{aligned} \Delta_{v^c} d_v^*(v^g, v^c) &= \frac{\pi_v^{2e}(v^g, G^2(v_2^c, v^g))}{\pi_u^{1e}(d^*(v^g, v_2^c), G^1(v_2^c, d^*(v^g, v_2^c)))} - \frac{\pi_v^{2e}(v^g, G^2(v_1^c, v^g))}{\pi_u^{1e}(d^*(v^g, v_1^c), G^1(v_1^c, d^*(v^g, v_1^c)))} \\ &= \frac{\pi_v^{2e}(v^g, G^2(v_2^c, v^g)) - \pi_v^{2e}(v^g, G^2(v_1^c, v^g))}{\pi_u^{1e}(d^*(v^g, v_2^c), G^1(v_2^c, d^*(v^g, v_2^c)))} \\ &\quad + \pi_v^{2e}(v^g, G^2(v_1^c, v^g)) \frac{\pi_u^{1e}(d^*(v^g, v_1^c), G^1(v_1^c, d^*(v^g, v_1^c))) - \pi_u^{1e}(d^*(v^g, v_2^c), G^1(v_2^c, d^*(v^g, v_2^c)))}{\pi_u^{1e}(d^*(v^g, v_1^c), G^1(v_1^c, d^*(v^g, v_1^c))) \pi_u^{1e}(d^*(v^g, v_2^c), G^1(v_2^c, d^*(v^g, v_2^c)))}. \end{aligned}$$

a solution to (23) (given $R^2 \in (0, 1)$) exists. It is unique, as $d^*(v, \cdot)$ is weakly decreasing for all and strictly decreasing for some v and $C_v^e(d^*(r, v^c), r) \leq 1$ for $v \leq v^*$ – thus the RHS crosses R^2 only once from above. This completes the proof for $R^1 + R^2 < 1$.

Abundant jobs If $R^1 + R^2 \geq 1$, then $C(u^c, v^c) = 0$. This implies that either u^c or v^c has to be equal to zero – more importantly, this implies that we can't define u^c as a function of v^c . I deal with this problem by defining the set $\Gamma^c = \{(u^c, v^c) : \min\{u^c, v^c\} = 0\}$, a new variable $a \in [-1, 1]$ and writing u^c and v^c as functions of a :

$$u^c(a) = \begin{cases} -a & \text{for } a \leq 0, \\ 0 & \text{for } a > 0, \end{cases} \quad v^c(a) = \begin{cases} 0 & \text{for } a \leq 0, \\ a & \text{for } a > 0. \end{cases}$$

Note that for any a , $(u^c(a), v^c(a)) \in \Gamma^c$ and for any $(u^c, v^c) \in \Gamma^c$ there exists a unique a , such that $(u^c(a), v^c(a)) = (u^c, v^c)$. Thus, if there exists a unique a that solves Equation (23), there also exists a unique (u^c, v^c) that solves it. Moreover, clearly $v^c(a)$ is continuous and increasing, whereas $u^c(a)$ is continuous and decreasing. Given that, existence and uniqueness follows from an analogous reasoning as above (for details, see the proof of Theorem 1 in Gola (2015)). \square

Lemma 3. The relation between the original and the modified set is as follows: (a) if ψ^e solves the modified set then its restriction to $[v^c, \sup\{v \in [v^c, 1] : \psi^e(v) \leq 1\}]$ solves the original one and (b) if a function $\psi(v) : [v^c, v^*] \rightarrow [u^c, u^*]$ solves the original set then we can always find its extension $\psi^e(v) : [v^c, 1] \rightarrow [u^c, 1 + B]$ that solves the modified one.

Proof. Note that if $v^* = \sup\{v \in [v^c, 1] : \psi^e(v) \leq 1\}$, then $\max\{\psi^e(v^*), v^*\} = 1$, as required. For $v \leq v^*$ we have that

$$\int_{v^c}^v C_v^e(\psi(r), r) dr \leq R^2,$$

$$C(\psi^e(v), v) - \int_{v^c}^v C_v^e(\psi(r), r) dr \leq R^1,$$

which means that the original and extended $C(\bullet)$, $C_v(\bullet)$, $\pi_u^1(\bullet)$ and $\pi_v^2(\bullet)$ are identical; and thus if (21) is met, (4) has to be met too. The equivalence of all other equations is trivial.

Claim (b) is trivial for $\psi(v^*) = 1$, as then ψ and ψ^e coincide. For $\psi(v^*) < 1$ the reasoning is slightly more complicated. Consider the following map: $T^1 : K \times [0, 1]^2 \rightarrow$

$[u^c, 1 + B]$:

$$(T^1d)(v, v^c, v^*) = \begin{cases} u^c & \text{for } v \in [0, v^c) \\ \psi(v) & \text{for } v \in [v^c, v^*] \\ 1 + \int_{v^*}^v \frac{\pi_v^2(t, G^2(v^*) + \int_{v^*}^t C^e(d(r), r) dr) + 2k(\frac{l^2}{R^2} C_v^e(d(t), t) + o_2)}{\pi_u^1(1, 1) + 2k(\frac{l^1}{R^1} C_u^e(d(t), t) + o^1)} dt & \text{for } v \in (v^*, 1]. \end{cases}$$

The restriction of the fixed point of this map to $[v^c, 1]$ solves the modified set of equations and is clearly an extension of $\psi(\cdot)$. By a reasoning analogous to that for $(Td)(v, v^v, u^c)$ in proof of Proposition 6, follows that there exists a unique fixed point of the map $(T^1d)(v, v^c, v^*)$. \square

Theorem 1 follows from Proposition 6 and Lemma 3 \square

C Changes in Status Structure

To simplify what follows, we need to first introduce new notation. The difference between the old and new values of any object O is denoted as $\Delta_\rho O$. The greater of the old and new values of any object O is denoted as $\max O$. Thus, for instance, the change in sector one size is $\Delta_\rho M^1$ and the greater critical ability in sector two is $\max v^c$.

Definition 10. Sector one surplus ($s^1(\bullet, \Theta)$) becomes *strictly more spread out* if $s_u^1(u, h, \Theta_s(\rho_1), \rho_2) > s_u^2(u, h, \Theta_s(\rho_1), \rho_1)$ for all $(u, h) \in [u^c(\rho_1), 1] \times [0, 1]$, where $\Theta_s(\rho_1)$ denotes the old stable assignment.

The Theorem below is the driving force of all my comparative statics results.

Theorem 3. If sector one surplus becomes strictly more spread out, then (i) $G^2(v, \rho_2) \geq G^2(v, \rho_1)$ for all v and holds strictly for $v \in [\max v^c(\rho), \max v^*(\rho)]$; (ii) $G^1(u, \rho_2) \leq G^1(u, \rho_1)$ for all u and holds strictly for $u \in [\max u^c(\rho), \max u^*(\rho)]$.

Proof of Theorem 3. This Theorem will be proved in a series of lemmas. I start, however, by defining two sets of sector two skill levels, which will be of crucial importance throughout the entire proof:

$$\begin{aligned} \Xi^0 &= \{v \in [\max v^c(\rho), \min v^*(\rho)] : \psi(v, \rho_2) = \psi(v, \rho_1) \wedge G^2(v, \rho_2) \leq G^2(v, \rho_1)\} \\ \Xi^1 &= \{v \in [\max v^c(\rho), \min v^*(\rho)] : \psi(v, \rho_2) \leq \psi(v, \rho_1) \wedge G^2(v, \rho_2) < G^2(v, \rho_1)\} \\ \Xi^2 &= \{v \in [\max v^c(\rho), \min v^*(\rho)] : \psi(v, \rho_2) \leq \psi(v, \rho_1) \wedge G^2(v, \rho_2) \leq G^2(v, \rho_1)\}. \end{aligned}$$

I also denote the total payoff sector i agent receives as $t^i(\cdot) = t^i(\cdot) + \tau^i(\cdot)$.

Lemma 4. A strict increase in the spread of sector one surplus implies that Ξ^2 is empty.

Proof of Lemma 4. Remember that $G_v^2(v) = \frac{\psi(v)C_{uv}(\psi(v),v)}{R^2}$. Take any $v_0 \in \Xi^0$ Note that by Equation (20) we have $\Delta_\rho G^1(\psi(v_0, \rho_1)) \geq 0$. Then we have:

$$\begin{aligned} \Delta_\rho t_u^1(\psi(v_0, \rho_1)) &= \Delta_\rho s_u^1(\psi(v_0, \rho_1), G^1(\psi(v_0, \rho_1), \Theta_s, \rho_2)) \\ &\quad + \int_{G^2(\psi(v_0, \rho_1), \rho_1)}^{G^2(\psi(v_0, \rho_1), \rho_2)} \pi_{uh}^1(\psi(v_0, \rho_1), r, \rho_1) dr > 0, \end{aligned}$$

as $\Delta_\rho s_u^1(u, h) > 0$ for any (u, h) , $\pi^1(\bullet)$ is supermodular and $\Delta_\rho G^1(\psi(v_0, \rho_1)) \geq 0$. Whereas for v_0 we have:

$$\Delta_\rho t_v^2(v_0) = \int_{G^2(v_0, \rho_1)}^{G^2(v_0, \rho_2)} \pi_{vh}^2(v_0, r) dr \leq 0,$$

as $\pi^2(\bullet)$ is supermodular and $\Delta_\rho G^2(v_0) \leq 0$. By differentiating Equation (6) wrt to v for both ρ_2 and ρ_1 , taking differences and rearranging, we arrive at:

$$\Delta_\rho \psi_v(v_0) = \frac{1}{t_u^1(\psi(v_0, \rho_1), \rho_2)} [\Delta_\rho t_v^2(v_0) - \psi_v(v_0, \rho_1) \Delta_\rho t_u^1(\psi(v_0, \rho_1))],$$

from which follows trivially that $\Delta_\rho \psi_v(v_0) < 0$.

Take any $v_1 \in \Xi^1$. Suppose that $\Delta_\rho \psi(v) \leq 0$ for all $v \in [v_1, \min v^*(\rho)]$, which implies that $v^*(\rho_2) > v^*(\rho_1)$. Remember that both for ρ_1 and ρ_2 we need to have $G^2(1) = 1$ and thus $\Delta_\rho G^2(1) = 0$. Therefore:

$$\begin{aligned} 0 &= \Delta_\rho G^2(v_1) + \frac{\int_{v_1}^{v^*(\rho_2)} C_v(\psi(v, \rho_2), v) dv - \int_{v_1}^{v^*(\rho_1)} C_v(\psi(v, \rho_1), v) dv - \Delta_\rho v^*(\rho)}{R^2} \\ &= \Delta_\rho G^2(v_1, \rho_1) - \int_{v_1}^{v^*(\rho_1)} \int_{\psi(v, \rho_2)}^{\psi(v, \rho_1)} \frac{C_{uv}(s, v)}{R^2} ds dv - \int_{v^*(\rho_2)}^{v^*(\rho_1)} \frac{1 - C_v(\psi(v, \rho_2), v)}{R^2} dv. \end{aligned}$$

Note that as $\psi(v, \rho_2) \leq 1$ it follows that $C_v(\psi(v, \rho_2), v) \leq 1$; hence we have that the two latter terms on the RHS are weakly and the first is strictly negative – contradiction. Therefore there needs to exist some $v \in (v_1, \min v^*(\rho)]$ such that $\Delta_\rho \psi(v) > 0$ for Ξ^1 to be non-empty. Denote the set of all such v 's as Ξ^3 ; then it follows that $\inf \Xi^3 \in \Xi^{031}$. This implies that $\Delta_\rho \psi_v(\inf \Xi^3) < 0$ which contradicts $v = \inf \Xi^3$. Thus, Ξ^1 has to be empty. Now consider any $v_2 \in \Xi^2$. Note that under $\Delta_\rho \pi_u^1(u, h) > 0$ for all (u, h) there has to exist some arbitrarily small $\epsilon > 0$ such that $v_2 + \epsilon < \min v^*(\rho)$ and for any $v \in (v_2; v_2 + \epsilon]$ we have $\Delta_\rho \psi(v) < 0$. This follows from continuity if $\Delta_\rho \psi(v_2) < 0$ and from the fact that if $\Delta_\rho \psi_v(v_2) = 0$ then $v_2 \in \Xi^0$ and $\Delta_\rho \psi_v(v_2) < 0$. Therefore, trivially, $\Delta_\rho G^2(v_2 + \epsilon) < 0$ and thus $v_2 + \epsilon \in \Xi^1$, which means that Ξ^2 has to be empty and concludes the proof. \square

Lemma 5. Suppose Ξ^2 is empty. Consider some $v_e \in [\max v^c(\rho), \min v^*(\rho)]$. Then $\Delta_\rho G^2(v_e) > 0$ implies (i) $\Delta_\rho G^2(v_e) > 0$ for all $v \in [v_e, \min v^*(\rho))$ and (ii) $G^2(\min v^*) \geq 0$.

³¹By continuity of $\Delta_\rho \psi(v)$, which follows from continuity of $\psi(v)$.

Proof. Suppose the (i) is false. Then the set $\Upsilon^3 = \{v \in [v_e, \min v^*(\rho)) : \Delta_\rho G^2(v) \leq 0\}$ has to be non-empty; but as $\Delta_\rho G^2(\cdot)$ is continuous in v ³², the non-emptiness implies that $v^1 = \min \Upsilon^3$ exists. Additionally, $v^1 > v_e$, as $\Delta_\rho G^2(v_e) > 0$. Define a new set $\Upsilon^4 = \{v \in [v_e, v^1] : \Delta_\rho \psi(v) \leq 0\}$ and $v^2 = \max \Upsilon^4$; by definition of v^1 , for any $v < v^1 \wedge v \in \Upsilon^4$ we have that $\Delta_\rho G^2(v) > 0$. As $[v_e, v^1]$ is a compact set and $\Delta_\rho \psi(v)$ is continuous v^2 won't exist only if Υ^4 is empty; but an empty Υ^4 implies that $\Delta_\rho \psi(v) > 0$ for any $v \in [v_e, v^1]$, which in turn means that $\Delta_\rho G^2(v^1) > 0$, which contradicts the definition of v^1 . Therefore v^2 needs to exist. Now suppose that $v^2 < v^1$; then we have $\Delta_\rho G^2(v^2) > 0$ and for any $v \in (v^2, v^1]$, $\Delta_\rho \psi(v) > 0$, which implies that

$$\Delta_\rho G^2(v^1) = \Delta_\rho G^2(v^2) + \frac{1}{R^2} \int_{v^2}^{v^1} \int_{\psi(r, \rho_1)}^{\psi(r, \rho_2)} C_{uv}(s, r) ds dr > 0$$

and also contradicts the definition of v^1 . Therefore it has to be the case that $v^1 = v^2$; but this implies that $\Delta_\rho(\psi(v^1)) \leq 0$ and $\Delta_\rho G^2(v^1) \leq 0$, which contradicts the emptiness of Ξ^2 . Claim (ii) follows from continuity of $G^2(\cdot)$. \square

Lemma 6. $\Delta_\rho G^2(\min v^*(\rho)) \geq 0$ implies that (i) for any $v > \min v^*(\rho)$ we have $\Delta_\rho G^2(v) \geq 0$ and (ii) for all $v \in [\min v^*(\rho), \max v^*(\rho))$ we have $\Delta_\rho G^2(v) > 0$.

Proof. Note that $\Delta_\rho G^2(\min v^*(\rho)) > (\geq) 0$ implies that $v^*(\rho_2) > (\geq) v^*(\rho_1)$ ³³. Thus, if $\Delta_\rho G^2(\min v^*(\rho)) = 0$ then $\min v^*(\rho) = \max v^c(\rho)$ and the second claim follows trivially. Whereas if $\Delta_\rho G^2(\min v^*(\rho)) > 0$ then $v^*(\rho_2) > v^*(\rho_1)$ and by the fact that all agents with $v \in (v^*, 1]$ join sector two for sure it follows that for $v \in (v^*(\rho_1), v^*(\rho_2))$ we also have $\Delta_\rho G^2(v) > 0$. Claim (i) for $v > \max v^*(\rho)$ follows easily from the aforementioned property of v^* . \square

Lemma 7. Empty Ξ^2 implies that $\Delta_\rho G^2(\max v^c(\rho)) > 0$.

Proof. Suppose not, which implies that $v^c(\rho_2) \geq v^c(\rho_1)$. Firstly, suppose that $\max v^c(\rho) \geq \min v^*(\rho)$. Then – as $v^*(\rho) > v^c(\rho)$ – it has to be that $v^*(\rho_2) > v^c(\rho_2) \geq v^*(\rho_1)$ and thus, by the same reasoning as in the proof of Proposition 6, $\Delta_\rho G^2(v^c(\rho_2)) > 0$, which implies $v^c(\rho_1) > v^c(\rho_2)$, contradiction. Thus, it has to be the case that $\max v^c(\rho) < \min v^*(\rho)$. $C(u^c(\rho_1), v^c(\rho_1)) = C(u^c(\rho_2), v^c(\rho_2))$ and thus $\Delta_\rho v^c(\rho) \geq 0$ implies $\Delta_\rho u^c(\rho) \leq 0$. As $\psi(v^c(\rho)) = u^c(\rho)$ and $\psi(v)$ is strictly increasing for any ρ we have: $\psi(v^c(\rho_2), \rho_1) \geq u^c(\rho_1)$,

³²Follows from continuity of $\psi(\cdot)$ and $C_v(\bullet)$.

³³ To see this, denote the ρ_i for which $v^*(\rho_i) = \max v^*(\rho)$ as ρ_m ; then, as $\Delta_\rho G^2(1) = 0$, we have:

$$0 = \Delta_\rho G^2(\min v^*(\rho), \rho_1) + \frac{1}{R^2} \int_{v^*(\rho_2)}^{v^*(\rho_1)} \frac{1 - C_v(\psi(v, \rho_m), v)}{R^2} dv.$$

As $1 - C_v(\psi(v, \rho_m), v) \geq 0$, the fact that $\Delta_\rho G^2(\min v^*(\rho)) > (\geq) 0$ implies that for this to hold we need $v^*(\rho_2) > (\geq) v^*(\rho_1)$.

$u^c(\rho_1) \geq u^c(\rho_2)$ and $u^c(\rho_2) = \psi(v^c(\rho_2), \rho_2)$, which trivially implies that:

$$\Delta_\rho \psi(v^c(\rho_2)) \leq 0.$$

But then $v^c(\rho_2) \in \Xi^2$ – contradiction. \square

Lemma 8. For all $v \in [\max v^c(\rho), \min v^*(\rho)]$, if $G^2(v, \rho_2) \geq (>)G^2(v, \rho_1)$ then $G^1(\psi(v, \rho_2), \rho_2) \leq (<)G^1(\psi(v, \rho_2), \rho_1)$.

Proof. From Equation (20), Definition 6 and Equation (7) follows that:

$$\begin{aligned} \Delta_\rho G^2(\psi(v, \rho_2)) &= -\frac{1}{R^1} R^2 \Delta_\rho G^2(v) \\ &\quad + \frac{1}{R^1} \left[\int_{\psi(v, \rho_1)}^{\psi(v, \rho_2)} C_u(r, v) dr - \int_{\psi(v, \rho_1)}^{\psi(v, \rho_2)} C_u(r, \psi^{-1}(r, \rho_1)) dr \right]. \end{aligned}$$

If $\psi(v, \rho_2) \geq \psi(v, \rho_1)$ then for any $r \in [\psi(v, \rho_1), \psi(v, \rho_2)]$, $\psi^{-1}(r, \rho_1) \geq v$ and my claim follows. If $\psi(v, \rho_2) < \psi(v, \rho_1)$ then for any $r \in [\psi(v, \rho_2), \psi(v, \rho_1)]$, $\psi^{-1}(r, \rho_1) < v$ and my claim follows as well. \square

Note that $\Delta_\rho G^2(\max v^c(\rho)) > 0$ trivially implies that $v < \max v^c(\rho)$, $\Delta_\rho G^2(v) \geq 0$. Thus, the results for sector two follow easily from Lemmas 4, 5, 6, 7 as well as continuity of $\Delta_\rho G^2(\cdot)$. As Lemma 6 has an exact sector one analogue, the sector one results follow from sector two results and Lemma 8. \square

C.1 Decrease in Sector One Wages

I will outline here sufficient conditions for an increase in the spread of sector one status to result in a decrease in sector one wages.

First of all, note that from the proof of Proposition 3 we have that any sector one wage increases by at most as much as C^1 . Thus, a fall in C^1 implies a fall in all sector one wages. In general, an increase in the spread of surplus has a negative, direct effect and a positive, general equilibrium effect on lowest status rewards. If the maximum spread of output in both sectors is low compared to k , then small changes to the status structure can result in strong general equilibrium effects and the overall effect on wages can be negative. To see this, consider the following example.

The output structure in both sectors is symmetric and output does not depend on firms. In particular, we have $\pi^1(u, h) = bu + k$ and $\pi^2(v, h) = bv + k$, where $b < \frac{k}{99}$. Suppose that local status does not matter in each sector, so that $l^1 = l^2 = 0$. Suppose further that only occupational prestige matters in both sectors initially ($p^1(\rho_1) = p^2 = 1$), but then the its weight falls in sector one ($p^1(\rho_2) = 1 - 99b$). Then from Equation (9) we have that $\psi(v, \rho_2) = \frac{1}{100}v + c_1$ and as none of Equations (4), (5), (10), (11) depends on

output, it follows that the new stable assignment does not depend on b (and neither does the old one, as the old problem is symmetric). The difference in the status reward of the least talented agents – a through that wage constant – does depend on b , however: the lower b , the higher the new status reward. In particular, for arbitrarily low b , the change in $\tau^1(v^c)$ is arbitrarily close to the increase in $\bar{u}k$. Thus, for arbitrarily low b wages fall for all sector one agents.

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